APPLICATION OF PARAMETER PLANE TECHNIQUES IN OSCILLATOR CIRCUITS

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by

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ABSTRACT

The parameter plane techniques were first introduced in an IEEE paper dated November 4, 1964. The paper dealt mainly with the theory of parameter planes whereby the roots of a polynomial could be determined graphically in terms of two parameters which may appear linearly in any of the coefficients.

Later on methods were developed in manipulating cases with parameters appearing as a product in the coefficients. Also by incrementing a third parameter every time, we can talk about a three-dimensional parameter space.

In this text parameter plane techniques have been used to plot the imaginary axis as a function of two parameters. From this plot we get curves of frequency versus each of the parameters so that we can study the sensitivity of different types of oscillators.

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1. Introduction

Up to now, oscillating circuits have been studied from the point of view of frequency changing as a function of one parameter of the circuit.

In this work the author attempts to study the frequency variation of oscillating circuits containing an active element when two parameters of the circuit or of the active element are continuously adjustable while, in certain cases, a third parameter is incremented.

Studying oscillators from this point of view is quite challenging because it is usually the case that in circuits like these more than one parameter can be changed.

So in the present text an attempt is made to adapt the newly developed parameter plane techniques to the study of oscillating circuits and at the same time we test the validity of this technique for circuits containing active elements.

The well known oscillators such as Phase Shift, Colpitt's, Hartley, Tuned plate and tuned grid are to be studied for different sets of parameters for each one and by incrementing the gain.

Evidently the same techniques can be applied to oscillating circuits with a transistor as an active element.

From the parameter plane curves we will derive the "sensitivity curves", that is, curves of frequency versus one of the two parameters.

2. Tube phase shift oscillator

In order to produce self-sustained oscillations in an amplifier, two conditions must be satisfied:

a. The voltage introduced from the output of the amplifier to its input must be in phase with the input voltage.

b. The overall amplification of the network must be equal to, or greater than unity.

That is if A = amplification parameter and B = fraction of output voltage of the amplifier introduced into the input of the amplifier, then $AB \geqslant 1$ and since this is a vector relationship,

$$A = |A| \angle \theta$$

and

$$B = |B| \angle \phi$$

Therefore relations to be satisfied are

$$|A| \cdot |B| = 1$$

$$\theta + \phi = 0$$

Let us examine and analyze the following phase-shift oscillator,

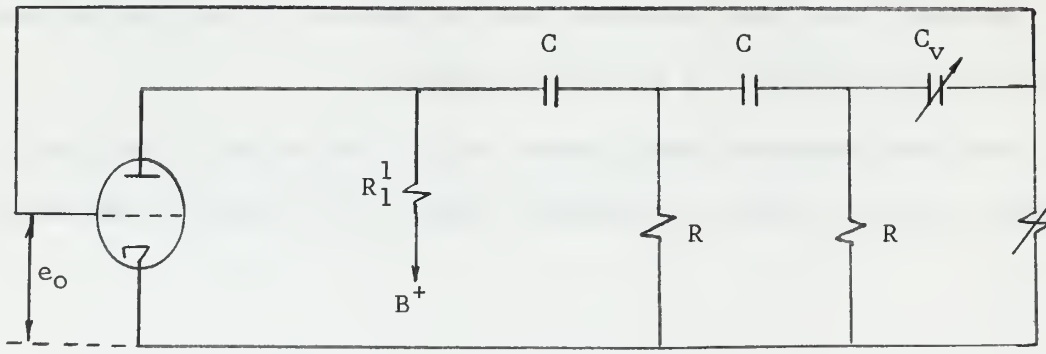


Figure 2-1. Phase Shift Oscillator

in which we fix the first two capacitors and resistors at the same value of C and R respectively, and make the last set of those, C_V, R_V variable, so that we will be able to study the frequency sensitivity from the parameter-plane point of view, with parameters C_v and R_v.

The equivalent circuit of the circuit in Figure 2-1 is as follows:

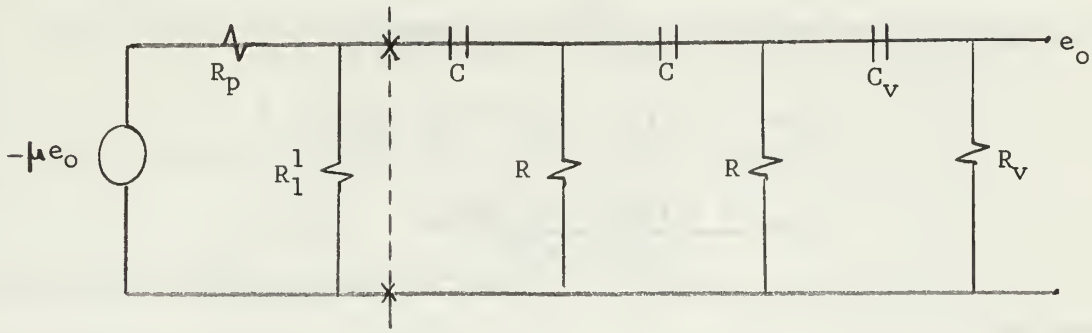


Figure 2-2. Equivalent Circuit

where R_p, μ are the plate resistance of the tube and amplification factor.

Using Thevenin's equivalent, looking to the left of points x-x we have:

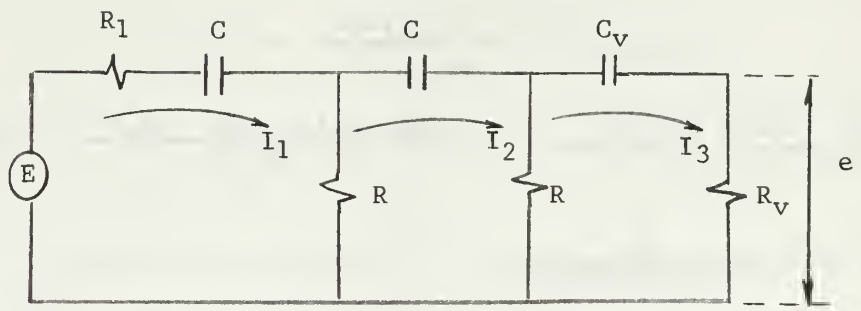


Figure 2-3. Equivalent Circuit

where

$$R_1 = \frac{R_i' R_p}{R_i' + R_p} \qquad E = -e_o \frac{R_i' \mu}{R_i' + R_p} = -e_o A$$

$$A = \frac{\mu R_i'}{R_i' + R_p}$$

2-1. Derivation of characteristic equation.

Setting up the 3 loop equations of figure 2-3

(1) $\qquad I_1(R_1 + R + jX) - I_2 R = E$

$$(2) \qquad -I_1R + I_2(2R + jX) - I_3R = 0$$

$$(3) \qquad -I_2R + I_3(R + R_v + jX_v) = 0$$

To solve for I_3, eliminate I_2 between (1) and (3)

$$(4) \qquad I_1(R_1 + R + jX) - I_3(R + R_v + jX_v) = E$$

$$I_1 = \frac{E + I_3(R + R_v + jX_v)}{R_1 + R + jX}$$

from (3)

$$I_2 = \frac{I_3(R + R_v + jX_v)}{R}$$

Put I_1, I_2 into (2)

$$(5) \qquad -R\,\frac{I_3(R + R_v + jX_v) + E}{R_1 + R + jX}$$

$$+ (2R + jX)\frac{I_3(R + R_v + jX_v)}{R} - I_3R = 0$$

$$-I_3\,\frac{R(R + R_v + jX_v)}{R_1 + R + jX} + I_3\,\frac{(2R + jX)(R + R_v + jX_v)}{R} - I_3R$$

$$= \frac{RE}{R_1 + R + jX}$$

$$I_3\left[-\frac{R(R + R_v + jX_v)}{R_1 + R + jX} + \frac{(2R + X)(R + R_v + jX_v)}{R} - R\right]$$

$$= \frac{RE}{R_1 + R + jX}$$

$$\frac{I_3}{R(R_1 + R + jX)}\Big[-R^2(R + R_v + jX_v) + (2R + jX)(R_1 + R + jX)\cdot$$

$$\cdot(R + R_v + jX_v) - R^2(R_1 + R + jX)\Big] = \frac{RE}{R_1 + R + jX}$$

$$\frac{I_3}{R}\Big[-R^2(R + R_v + jX_v) + (2R + jX)(R_1 + R + jX)(R + R_v + jX_v)$$

$$-R^2(R_1 + R + jX)\Big] = RE$$

(6) $$I_3 = \frac{R^2E}{-R^2(R+R_v+jX_v)+(2R+jX)(R_1+R+jX)(R+R_v+jX_v) - R^2(R_1+R+jX)}$$

The voltage at the output of the phase circuit is

$$e = I_3R_v,$$

so calling the denominator of (6), D

(7) $$e = I_3R_v = \frac{R^2R_vE}{D}$$

The total amplification of circuit $AB = \frac{e}{e_o}$ and from (7)

$$\frac{R^2R_v}{D} = \frac{e}{E} \quad \frac{e}{-Ae_o} = (-\frac{1}{A})\,\frac{e}{e_o} = (-\frac{1}{A})(AB)$$

$$= -B$$

so

(8) $$B = -\frac{R^2R_v}{D}$$

The characteristic equation of this feedback amplifier is $AB = 1$

or

$$AB = A(-\frac{R^2R_v}{D}) = 1$$

or

$$AR^2R_v + D = 0$$

which by appropriate manipulation leads to

(9) $$C^2\Big[C_vR_v(AR^2 + R^2 + 2RR_1) + C_v(R^2R_1)\Big]\,S^3 + \Big[R_vC_v(3RC + R_1C)$$

$$+ C_v(2R^2C + RR_1C) + (R^2C^2 + 2RR_1C^2)\Big]$$

$$+\left[R_V C_V + C_V R + 3RC + R_1 C\right] S + 1 = 0$$

2-2. Summary of parameter plane techniques.

Equation (9) is a third order equation for which we apply the parameter-plane techniques for variables R_V and C_V. Those variables occur in the coefficients of the equation as follows:

$$\left[R_V C_V K_1 + C_V K_2\right] S^3 + \left[R_V C_V K_3 + C_V K_4 + K_5\right] S^2$$

$$+\left[R_V C_V + C_V K_6 + K_7\right] S + 1 = 0.$$

In other words some of the coefficients of the characteristic equation appear as

$$a_K = b_K C_V + C_K R_V + h_K C_V R_V + d_K$$

Such a characteristic equation is treated by Dr. G.J. Thaler as follows: Let characteristic equation

$$F(S) = \sum_{K=0}^{n} a_K S^K = 0$$

where

$$a_K = b_K C_V + C_K R_V + h_K C_V R_V + d_K$$

If

$$S = -\zeta\omega + j\omega\sqrt{1 - \zeta^2} = \omega(-\zeta + j\sqrt{1 - \zeta^2}) = \omega(\cos\theta +$$

$$j\sin\theta) = \omega e^{j\theta}$$

where

$$\theta = \cos^{-1}\zeta$$

Then

$$S^K = \omega^K e^{jK\theta} = \omega^K(\cos K\theta + j\sin K\theta)$$

But Chebyshev functions

$$T_K(\zeta) = \cos K\theta = \cos(k\cos^{-1}\zeta)$$

$$\overline{U}_K(\zeta) = \frac{\sin K\theta}{\sin\theta}$$

So

$$S^K = \omega^K\left[(-1)^K T_K(\zeta) + j\sqrt{1-\zeta^2}\,(-1)^{K+1}\,U_K(\zeta)\right]$$

Making real and imaginary parts equal to zero and knowing that

$$T_K(\zeta) = \zeta\,\overline{U}_K(\zeta) - \overline{U}_{K-1}(\zeta)$$

$$(10)\qquad \begin{aligned} &\sum_{K=0}^{n} (-1)^K a_K\,\omega^K\,\overline{U}_{K-1}(\zeta) = 0 \\ &\sum_{K=0}^{n} (-1)^K a_K\,\omega^K\,\overline{U}_K(\zeta) = 0 \end{aligned}$$

Equations (10) for the given problem may be reformed:

$$(11)\qquad \begin{aligned} &C_vB_1(\zeta,\omega) + R_vC_1(\zeta,\omega) + C_vR_vH_1(\zeta,\omega) + D_1(\zeta,\omega) = 0 \\ &C_vB_2(\zeta,\omega) + R_vC_2(\zeta,\omega) + C_vR_vH_2(\zeta,\omega) + D_2(\zeta,\omega) = 0 \end{aligned}$$

where

$$B_1 = \sum_{K=0}^{n} (-1)^K b_K\,\omega^K T_K(\zeta) \qquad B_2 = \sum_{K=0}^{n} (-1)^K b_K\,\omega^K U_K(\zeta)$$

$$C_1 = \sum_{K=0}^{n} (-1)^K c_K\,\omega^K T_K(\zeta) \qquad C_2 = \sum_{K=0}^{n} (-1)^K c_K\,\omega^K U_K(\zeta)$$

$$H_1 = \sum_{K=0}^{n} (-1)^K h_K\,\omega^K T_K(\zeta) \qquad H_2 = \sum_{K=0}^{n} (-1)^K h_K\,\omega^K U_K(\zeta)$$

$$D_1 = \sum_{K=0}^{n} (-1)^K d_K\,\omega^K T_K(\zeta) \qquad D_2 = \sum_{K=0}^{n} (-1)^K h_K\,\omega^K U_K(\zeta)$$

Solution of equations (11) for C_v, R_v give:

$$(C_v)_K = \frac{1}{2\Delta_{BH}}\left[-(\Delta_{DH} + \Delta_{BC}) + (1)^K R_{C_v}\right]$$

(12)

$$(R_v)_K = \frac{1}{2\Delta_{CH}}\left[-(\Delta_{DH} + \Delta_{CB}) + (1)^{K+1} R_{R_v}\right]$$

where

$$\Delta_{BC} = \begin{vmatrix} B_1 & B_2 \\ C_1 & C_2 \end{vmatrix} \text{ etc.}$$

and

$$R_{C_v}^2 = (\Delta_{BC} + \Delta_{DH})^2 - 4\Delta_{BH}\Delta_{DC} = R_{R_v}^2 \equiv R_{CR}$$

To plot the imaginary axis of S plane (ζ = 0) on the parameter plane, we set ζ = 0 to equation (12) and for every value of frequency ω we get a pair of C_v, R_v from equation (12), for the positive sign of R_{C_v} or R_{R_v} and a pair for the negative sign of R_{C_v} or R_{R_v} which come from the extraction of the square root.

If we do the same thing for different values of gain A we shall get a family of curves ζ = 0 for different A which may represent different tubes.

Then if we fix R_v and use A and C_v as parameters we can get the ζ = 0 curves.

Let us give some numerical values to the fixed parameters of the circuit, such as: R = 1M and R_1 = 10K . Now for different typical values for the gain A = 30, 40, 50 and for capacitors C = 10^{-6}, 10^{-9}, 10^{-12} fd. We study the result of the oscillator keeping as parameters, C_v and R_v.

2-3. Parameter plane curves C_v versus R_v for different ranges of frequencies.

A = 30, 40, 50

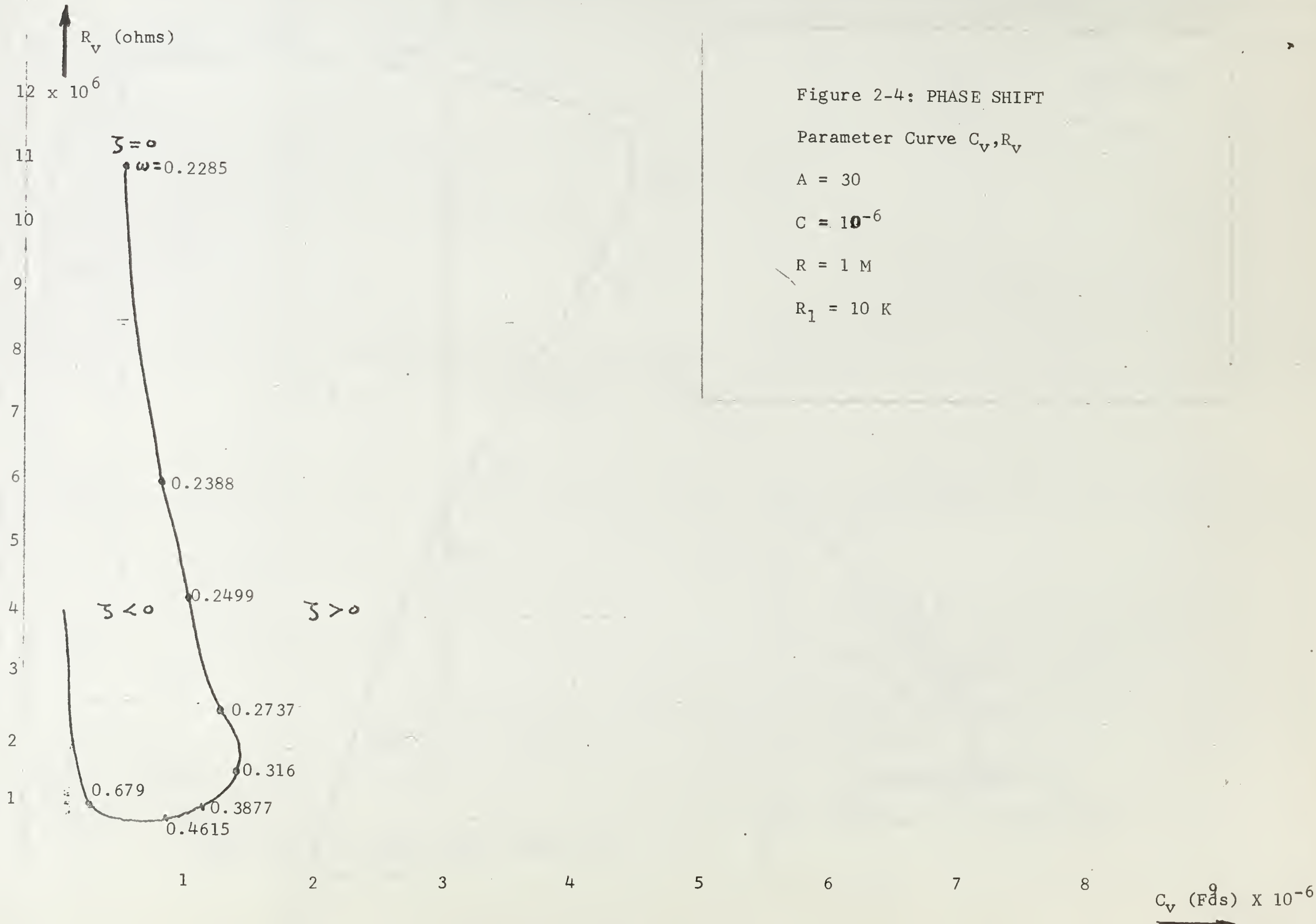


Figure 2-4: PHASE SHIFT

Parameter Curve C_v, R_v

$A = 30$

$C = 10^{-6}$

$R = 1$ M

$R_1 = 10$ K

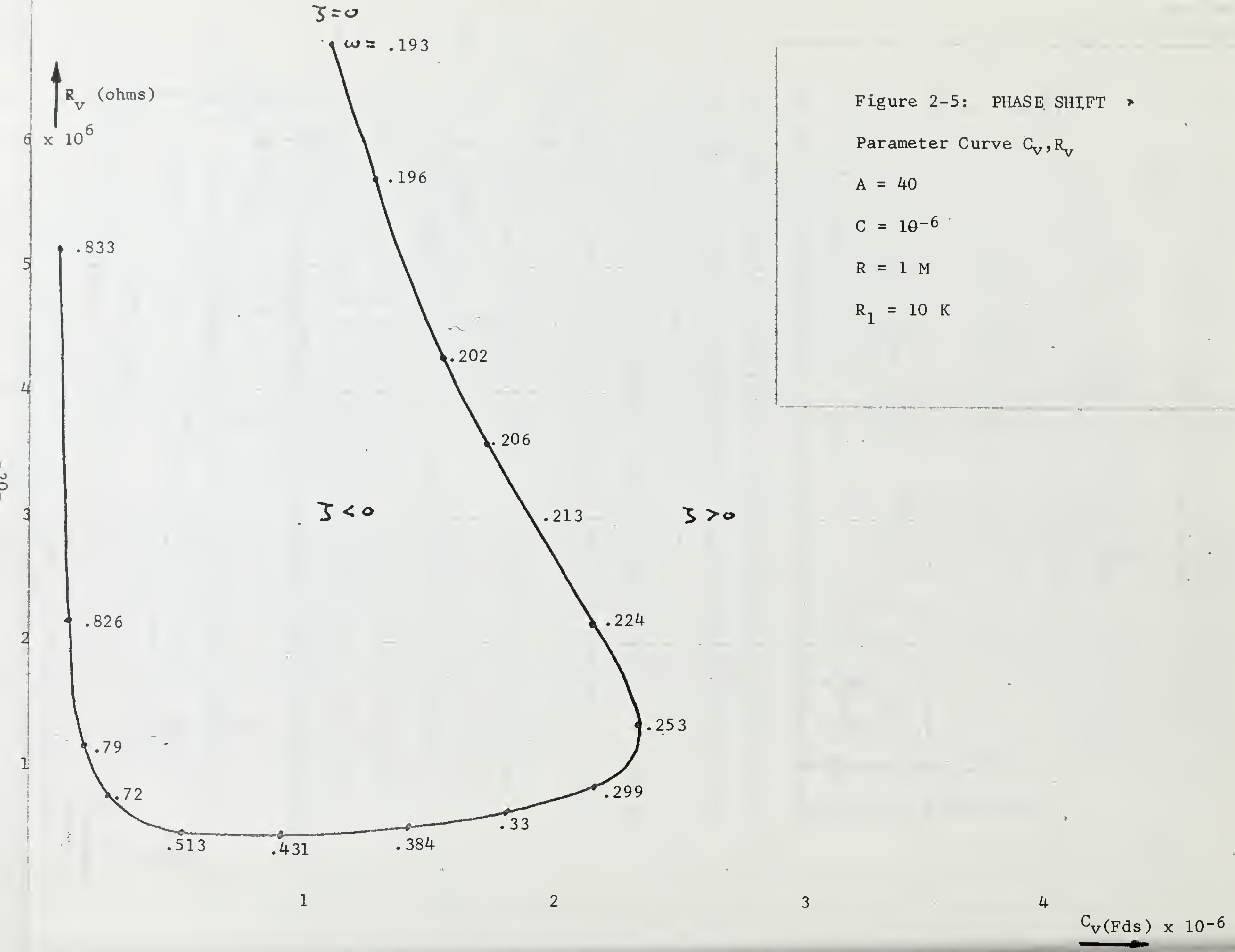


Figure 2-5: PHASE SHIFT

Parameter Curve C_v, R_v

$A = 40$

$C = 10^{-6}$

$R = 1$ M

$R_1 = 10$ K

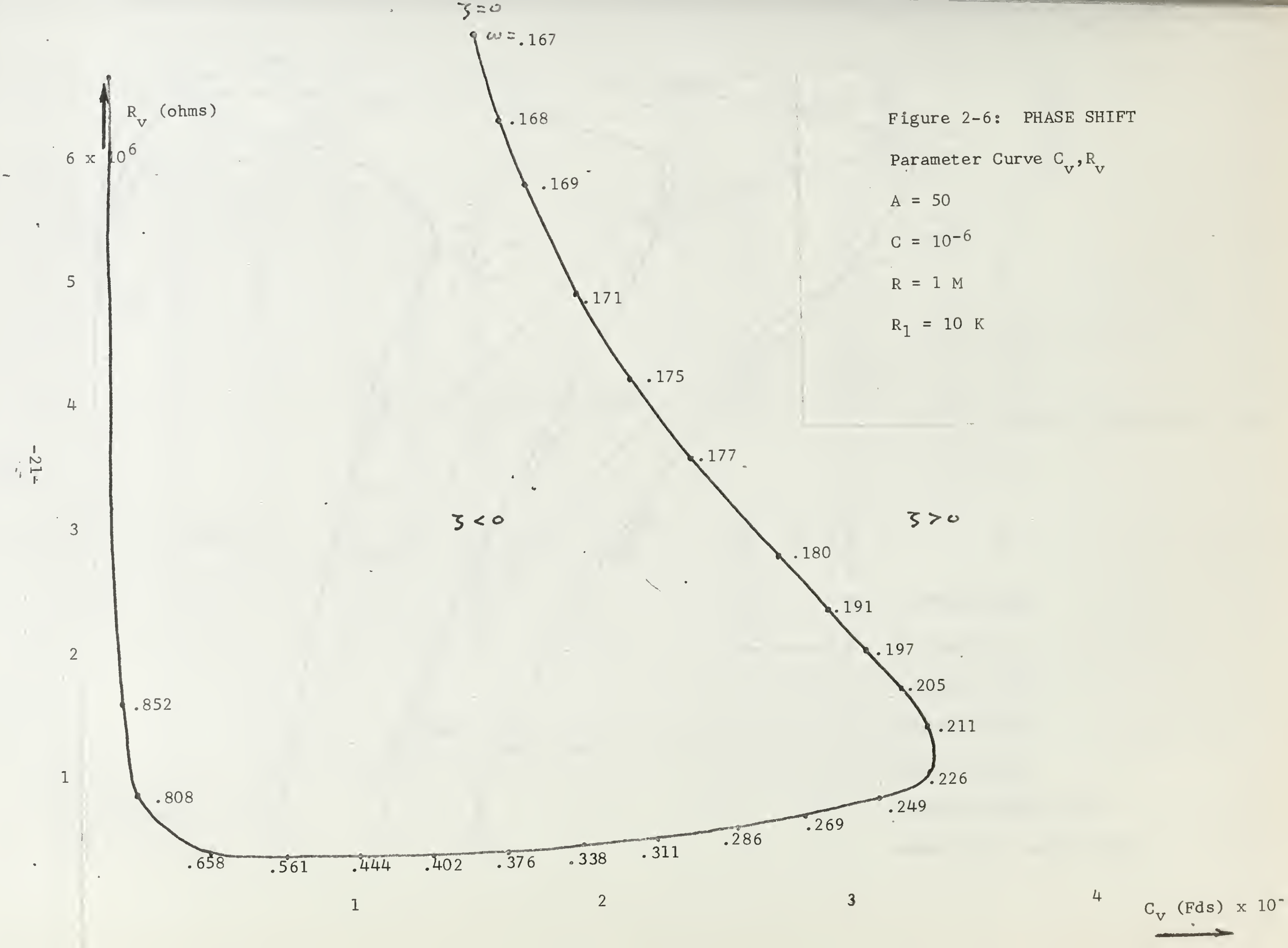


Figure 2-6: PHASE SHIFT

Parameter Curve C_v, R_v

$A = 50$

$C = 10^{-6}$

$R = 1$ M

$R_1 = 10$ K

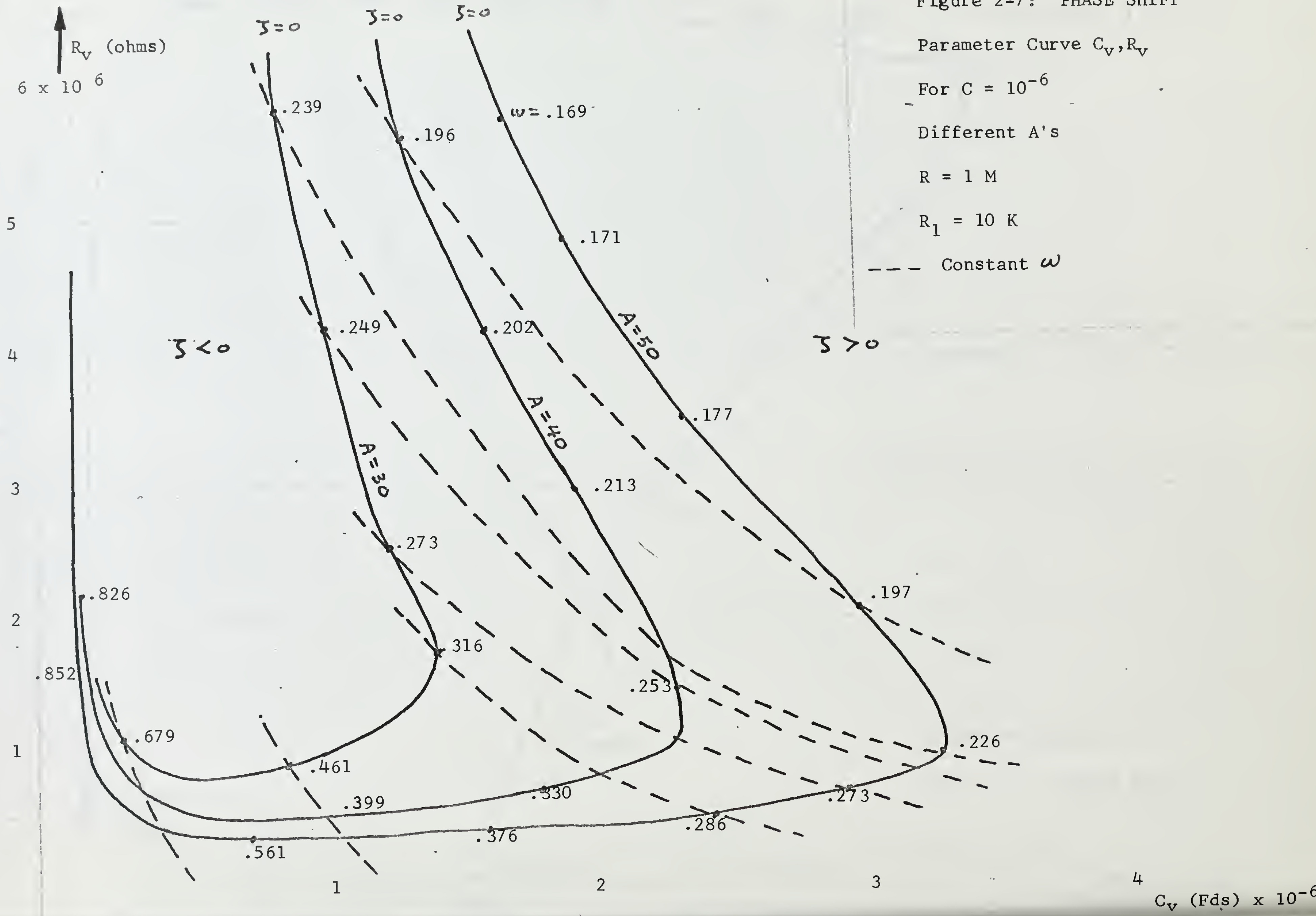


Figure 2-7: PHASE SHIFT

Parameter Curve C_V, R_V

For $C = 10^{-6}$

Different A's

R = 1 M

R_1 = 10 K

--- Constant ω

$$R_1 = 10K$$

$$C = 10^{-6}\ f$$

$$R = 1\ M$$

<u>For A = 30</u>

$$\left[31.02\ R_vC_v + 10^4C_v\right] S^3 + \left[3.01\ R_vC_v + 2.01 \times 10^6C_v + 1.02\right] S^2$$

$$+ \left[R_vC_v + 10^6C_v + 3.01\right] S + 1 = 0$$

<u>For A = 40</u>

$$\left[41.02R_vC_v + 10^4C_v\right] S^3 + - - - \text{(same as above)}$$

<u>For A = 50</u>

$$\left[51.02R_vC_v + 10^4C_v\right] S^3 + - - - \text{(same as above)}$$

<u>Parameters C_v, R_v</u> <u>$C = 10^{-9}$</u>

$$A = 30,\ 40,\ 50$$

$$R_1 = 10\ K$$

$$R = 1\ M$$

<u>For A = 30</u>

$$\left[C_vR_v(0.3102 \times 10^{-4}) + C_c(0.1 \times 10^{-1})\right] S^3 +$$

$$\left[C_vR_v(0.301 \times 10^{-2}) + C_v(0.201 \times 10^4) + (0.102 \times 10^{-5})\right] S^2 +$$

$$\left[R_vC_v + C_v(0.1 \times 10^7) + (0.301 \times 10^{-2})\right] S + 1 = 0$$

<u>For A = 40</u>

$$\left[C_vR_v(0.4102 \times 10^{-4}) + C_v(0.1 \times 10^{-1})\right] S^3 + - - \text{(same as above)}$$

<u>For A = 50</u>

$$\left[C_vR_v(0.5102 \times 10^{-4}) + C_v(0.1 \times 10^{-1})\right] S^3 + - - \text{(same as above)}$$

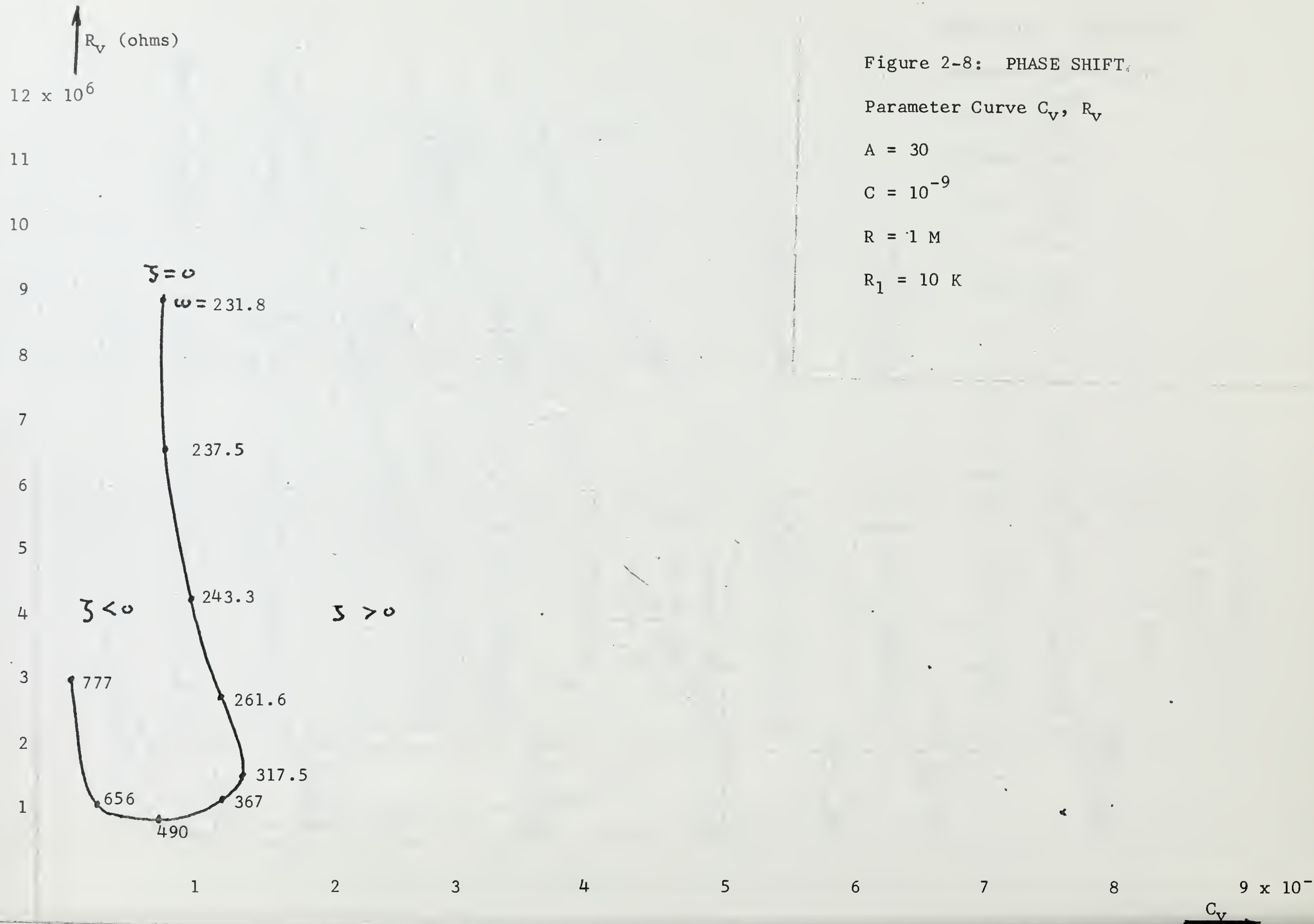


Figure 2-8: PHASE SHIFT

Parameter Curve C_V, R_V

$A = 30$

$C = 10^{-9}$

$R = 1$ M

$R_1 = 10$ K

Figure 2-9: PHASE SHIFT

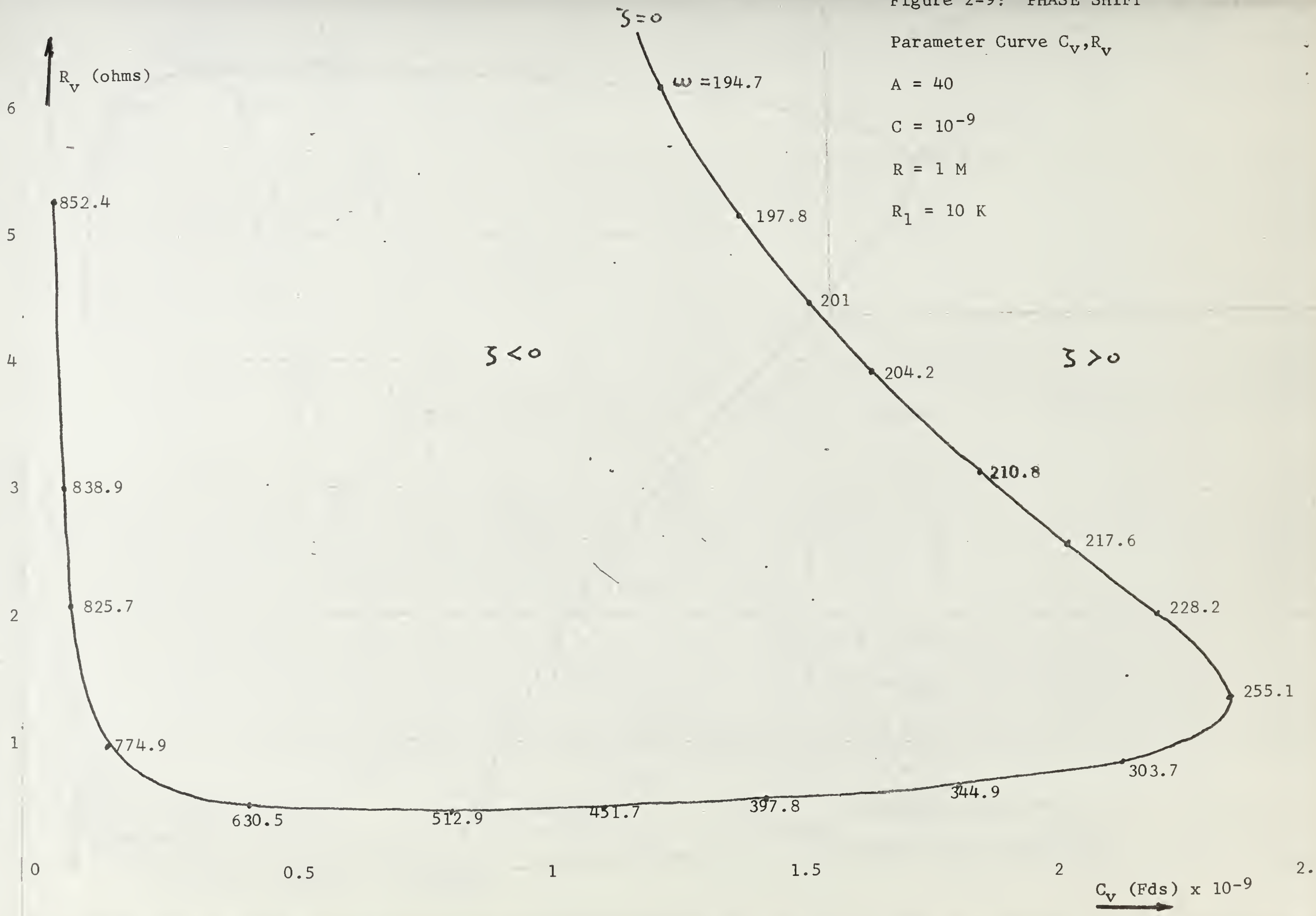
Parameter Curve C_v, R_v

$A = 40$

$C = 10^{-9}$

$R = 1$ M

$R_1 = 10$ K



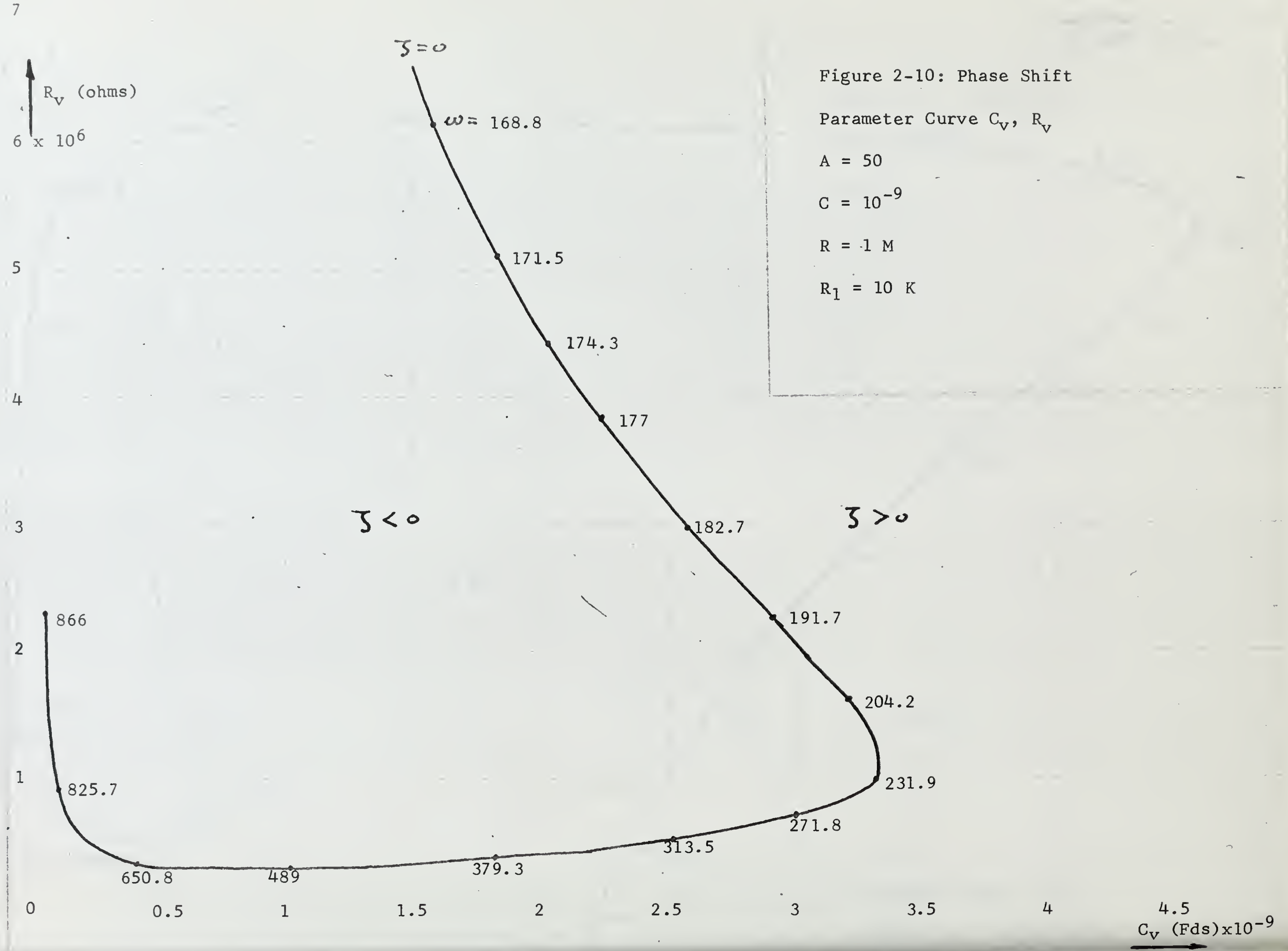


Figure 2-10: Phase Shift Parameter Curve C_v, R_v

$A = 50$

$C = 10^{-9}$

$R = 1$ M

$R_1 = 10$ K

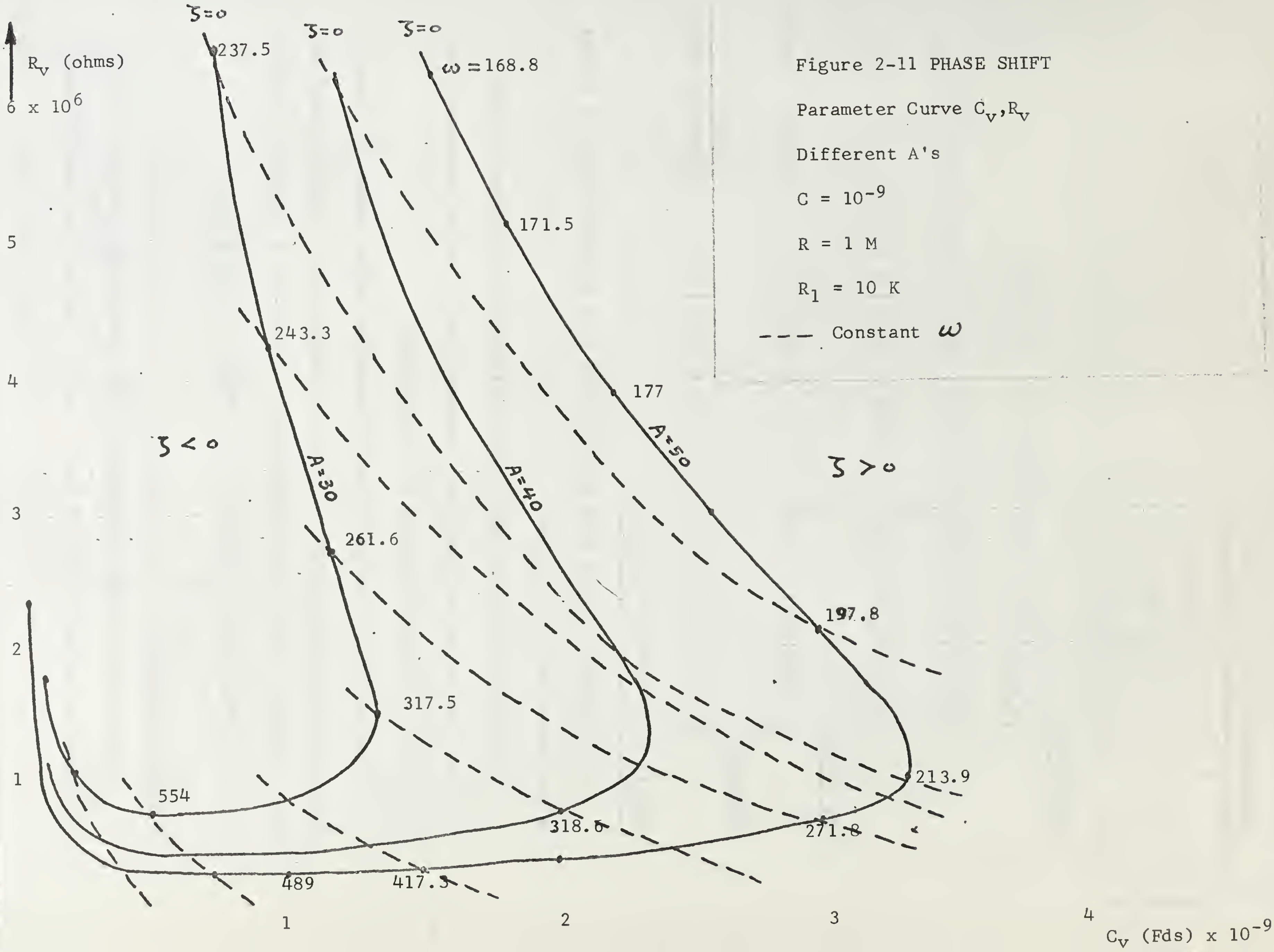


Figure 2-11 PHASE SHIFT

Parameter Curve C_v, R_v

Different A's

$C = 10^{-9}$

$R = 1$ M

$R_1 = 10$ K

--- Constant ω

Parameters C_v, R_v $C = 10^{-12}$

$A = 30, 40, 50$

$R_1 = 10\ k$

$R = 1\ M$

For A = 30

$$\left[C_vR_v(0.3102 \times 10^{-10}) + C_v(0.1 \times 10^{-7})\right] S^3 +$$

$$\left[C_vR_v(0.301 \times 10^{-5}) + C_v(0.201 \times 10^{1}) + (0.102 \times 10^{-11}\right] S^2 +$$

$$\left[C_vR_v + C_v(0.1 \times 10^{7}) + (0.301 \times 10^{-5})\right] S + 1 = 0$$

For A = 40

$$\left[C_vR_v(0.4102 \times 10^{-10}) + C_v(0.1 \times 10^{-7})\right] S^3 + - - -\text{(same as above)}$$

For A = 50

$$\left[C_vR_v(0.5102 \times 10^{-10}) + C_v(0.1 \times 10^{-7})\right] S^3 + - - -\text{(same as above)}$$

Results for Parameters C_v, R_v

The computer results have been plotted into different sets of curves on the parameter C_v, R_v plane.

The capacitors C have been given extreme and mid-values in order to study the pattern of oscillator behavior and it is not necessary that this type of analysis hold for all frequencies.

This analysis is valid for mid-frequencies range, where the inter-electrode capacitance effects of active elements can be neglected and the active element gain can be considered as a function of passive elements only and independent of frequencies. Namely $A = \dfrac{\mu R_i'}{R_i' + R_p}$.

For Constant Gain A

Studying the curves we recognize that for the same value of C and for different values of gain A we can control the oscillation frequency

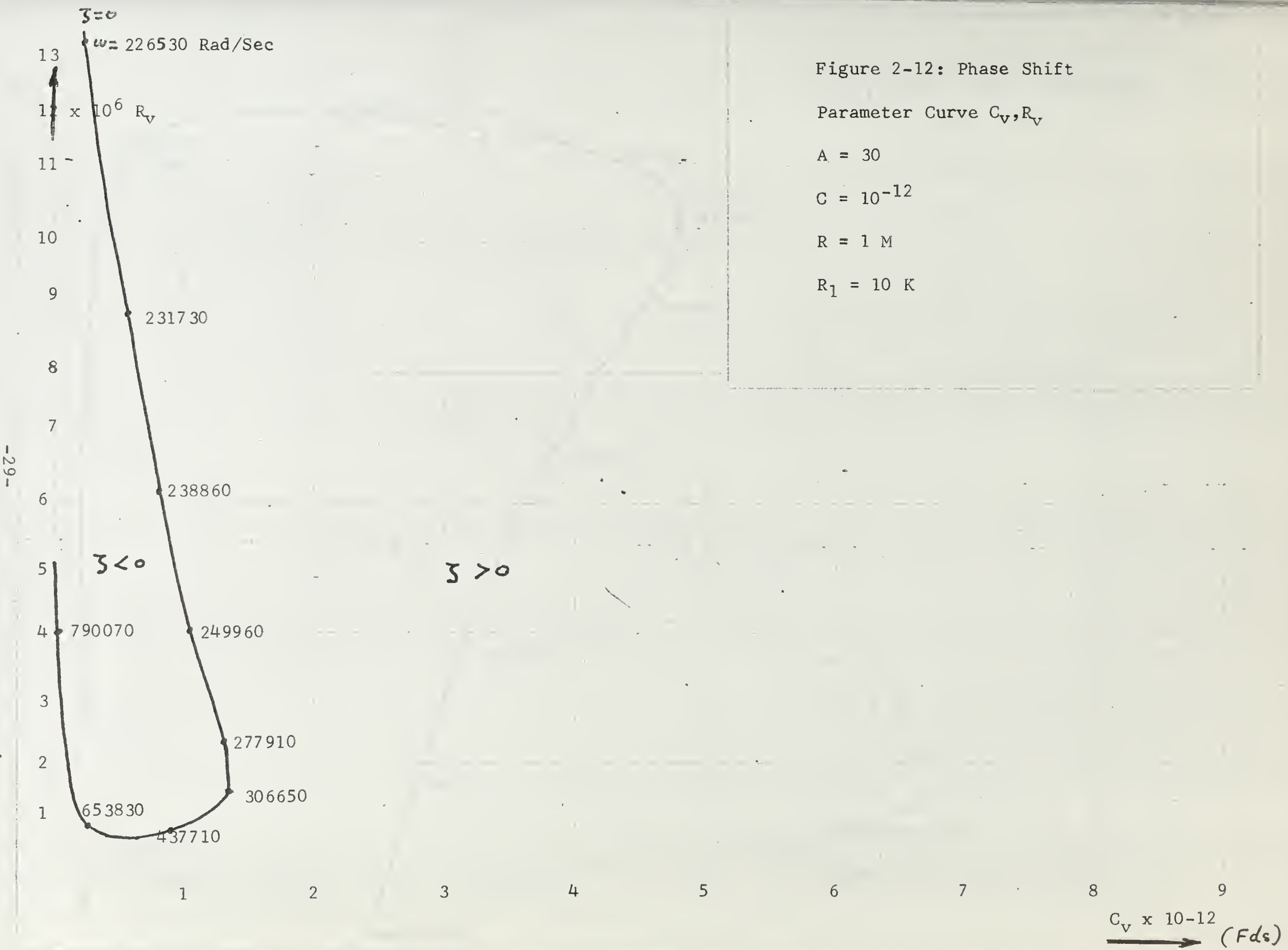


Figure 2-12: Phase Shift

Parameter Curve C_v, R_v

$A = 30$

$C = 10^{-12}$

$R = 1\ M$

$R_1 = 10\ K$

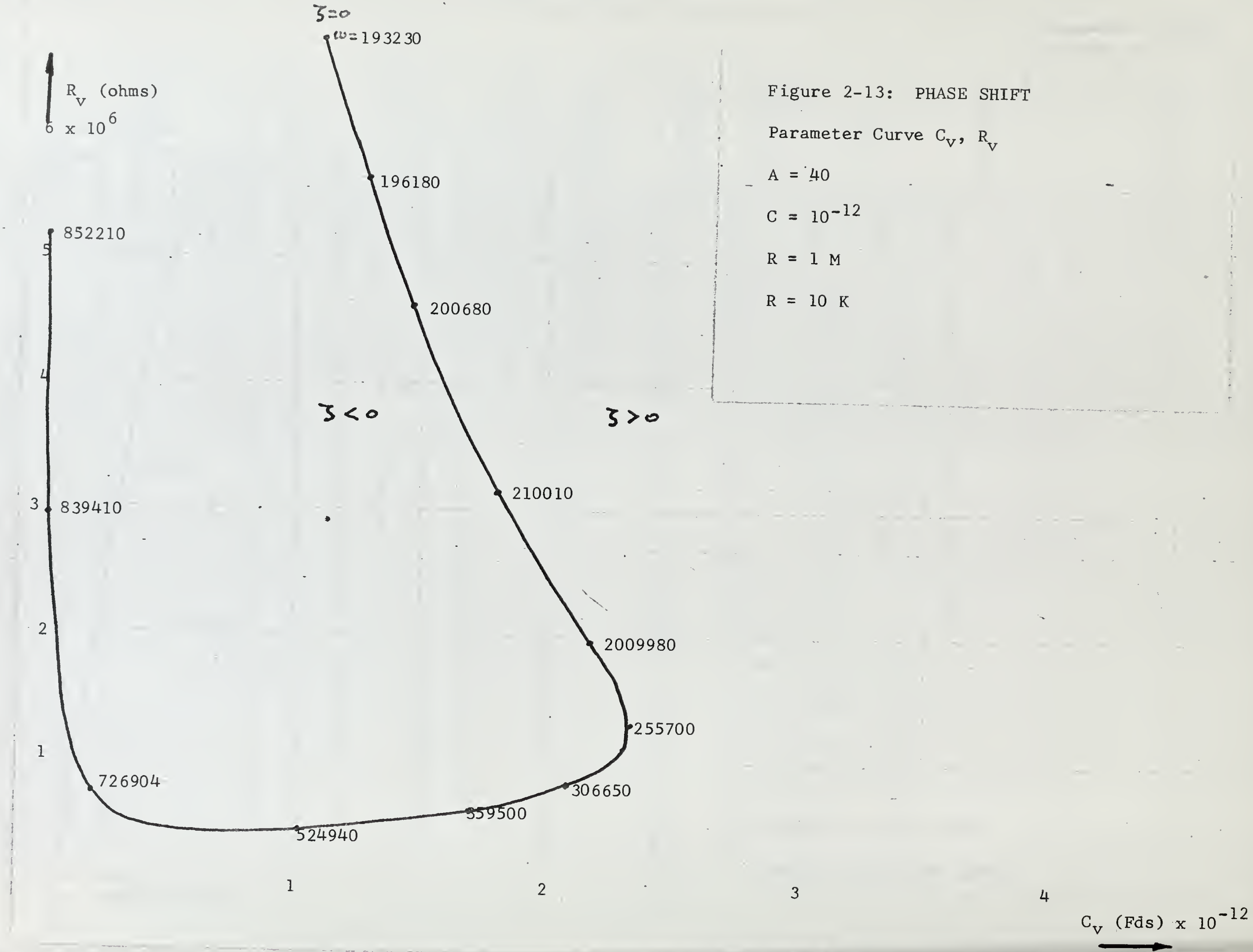


Figure 2-13: PHASE SHIFT

Parameter Curve C_V, R_V

$A = 40$

$C = 10^{-12}$

$R = 1$ M

$R = 10$ K

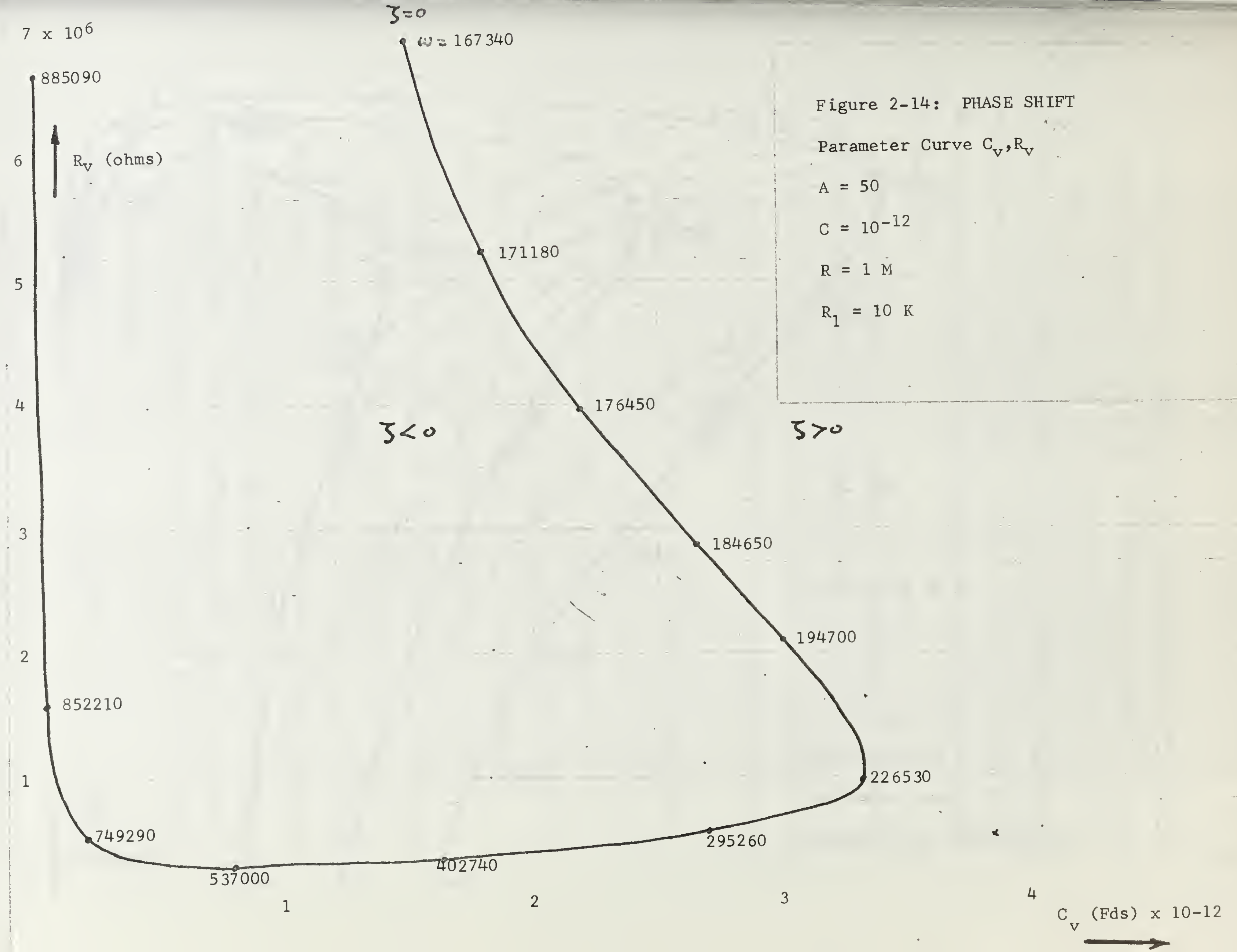


Figure 2-14: PHASE SHIFT

Parameter Curve C_v,R_v

A = 50

C = 10^{-12}

R = 1 M

R_1 = 10 K

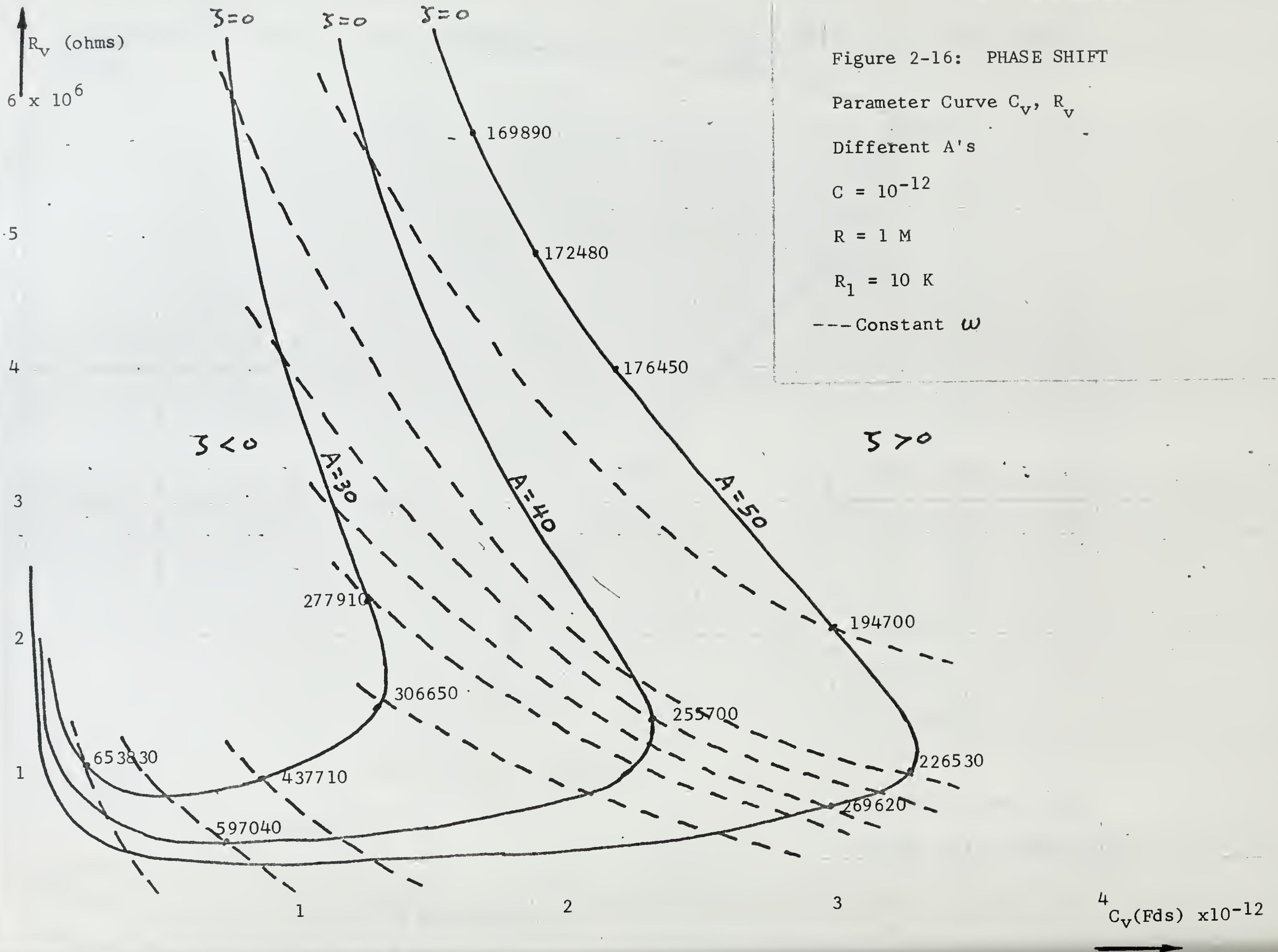


Figure 2-16: PHASE SHIFT

Parameter Curve C_v, R_v

Different A's

$C = 10^{-12}$

$R = 1$ M

$R_1 = 10$ K

via C_v, R_v over a certain range of frequencies depending on the value of C. For lower values of C the range of frequencies is increased and vice versa. For example, for gain A = 50 and $C = 10^{-12}$, we control frequency from ω = 169890 to 885090 rad/sec while for A = 50 and $C = 10^{-6}$ we can vary ω from .168 to .885 as shown in Figures 2-14 and 2-6 respectively. We can roughly say that the range of frequencies spans about one decade of frequencies.

For one value of R_v there are usually 2 values of C_v and therefore 2 different frequencies, one at the low part and one at the high part of the frequency band, that the oscillator can oscillate. For example, from Figure 2-5, for R_v = 2.2 M correspond ω = .826 rad/sec C_v = .05 μf and ω = .224 rad/sec C_v = 2.15 μf.

Also for a certain part of the band, for a fixed value at C_v there are 2 values of R_v that can make the oscillator work, one at mid-part and one at low part of the frequency band, i.e. from Figure 2-10 for C_v = 2 μf we have ω = 356 rad/sec, R_v = 0.48 M in the mid-part and ω = 174.3 rad/sec, R_v = 4.48 M in the low part of the frequency band.

The low part of the band presents the more linear variation of parameters and may be considered that it gives more stable frequency results, such as for ω = 193230 to ω = 240670 rad/sec of Figure 2-13.

The mid-part is almost horizontal, like the part between frequencies ω = 359500 and ω = 620080 of Figure 2-14, so small variations of R_v may cause drift from constant amplitude oscillations to increasing or decreasing in amplitude. So that decreasing in amplitude may cause stop of oscillation and increasing of amplitude will eventually cause saturation of active elements.

Also, variation at C_v, if R_v remains constant, will cause a change in frequency so this effect may be used to measure variations of C_v by measuring the frequency change.

The high part of the band, like from ω = 814370 to ω = 885090 rad/sec of Figure 2-14, is almost vertical and small variations of C_v may eventually cause drift to right or left of $j\omega$ axis in the S plane and hence have an increase or decrease in amplitude of oscillations.

For various values of gain A

If gain A is increased, say from A = 30 to A = 50, the curves of $\zeta = 0$ give the same pattern of frequency variation, namely, low, mid and high part of the frequency band.

From Figure 2-9, the low part is from about ω = 194.7 to ω = 255.1 rad/sec, mid part from 225.1 to 774.9 rad/sec and the high part from 774.9 to 852.4 rad/sec.

The lower the gain is, the narrower the band of frequencies is. For example, from Figure 2-11 for A = 30, the frequency band is about from ω = 237.5 to ω = 740 rad/sec while for A = 50 the band ranges from ω = 168.8 to ω = 866 rad/sec.

The lower the gain A, the narrower the range of values that C_v can take, while R_v can be varied almost over the same range of values for all gains, i.e. from Figure 2-7 for A = 30, C_v varies from 0.1 to 1.4 μf and for A = 50, C_v varies from 0.05 to 3.3 μf. R_v varies for all gains almost between 0.5 to 7 M .

R_v has a minimum value that the system can oscillate, and this value goes down as A goes up. As shown in Figure 2-16 for A = 30, minimum R_v = 0.8 M , for A = 50 minimum R_v = 0.35 M .

C_v has a maximum possible value that oscillations can be sustained. This maximum value is increased as gain A goes up. From Figure 2-16, for A = 30, maximum $C_v = 1.35\ \mu\mu f$ and for A = 50, maximum $C_v = 3.3\ \mu\mu f$.

2-4. Parameter plane curves C_v versus A for different ranges of frequencies.

Now we wish to study the problem from a different point of view. We want to see the frequency variation pattern and frequency sensitivity, by plotting the $\zeta = 0$ curves with variable parameters C_v and A.

Typical values for the circuit constants can be given such as:

$R_1 = 10\ K$

$R = R_v = 1\ M$

and also for the capacitors C we give values from 10^{-6} to 10^{-12} fd.

For $C = 10^{-6}$ equation (9) becomes

$$C^2\left[C_v AR^2R_v + C_v(R_vR^2 + 2R_vR_1R + R^2R_1)\right]S^3$$

$$+\left[C_v(3RCR_v + R_1R_vC + 2R^2C + RR_1C) + (R^2C^2 + 2RR_1C^2)\right]S^2$$

$$+\left[C_v(R + R_v) + (3RC + R_1C)\right]S + 1 = 0$$

putting numerical values

$$\left[10^6\ C_vA + 1.03 \times 10^{18}\ C_v\right]S^3$$
$$+\left[5.03 \times 10^6\ C_v + 1.02\right]S^2 + \left[2 \times 10^6\ C_v + 3.01\right]S + 1 = 0$$

For $C = 10^{-9}$

$$\left[C_vA + C_v\ (1.03)\right]S^3 + \left[C_v\ (5.03 \times 10^3) + (1.02 \times 10^{-6})\right]S^2$$

$$+\left[C_v(2 \times 10^6) + (3.01 \times 10^{-3})\right]S + 1 = 0$$

For $C = 10^{-12}$

$$\left[C_vA(10^{-6}) + C_v(1.03 \times 10^{-6})\right]S^3 + \left[C_v(5.02) + 1.02 \times 10^{-12}\right]\cdot$$

$$\cdot S^2 + \left[C_v(2 \times 10^6) + 3.01 \times 10^{-6}\right]S + 1 = 0$$

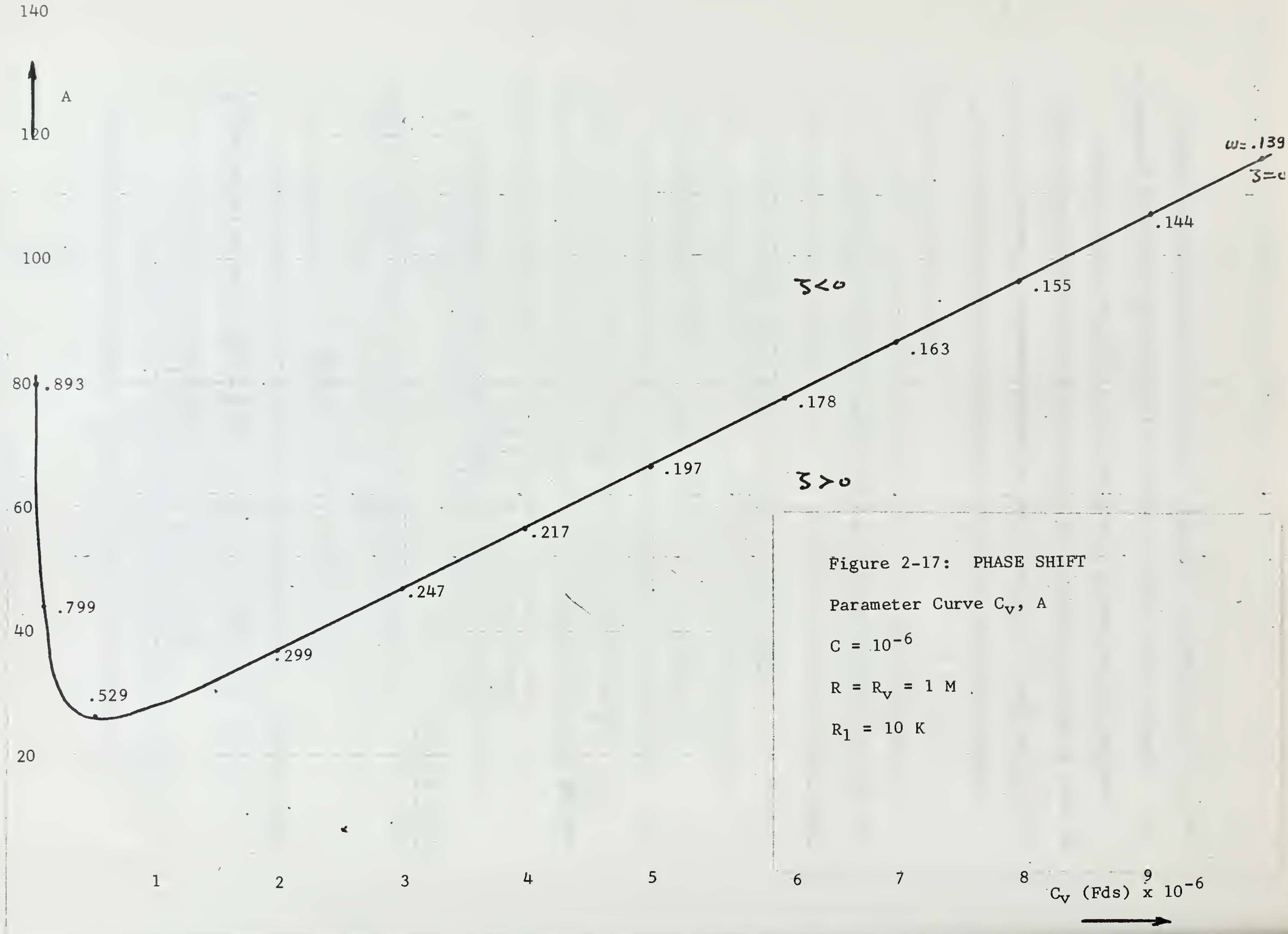


Figure 2-17: PHASE SHIFT

Parameter Curve C_v, A

$C = 10^{-6}$

$R = R_v = 1$ M

$R_1 = 10$ K

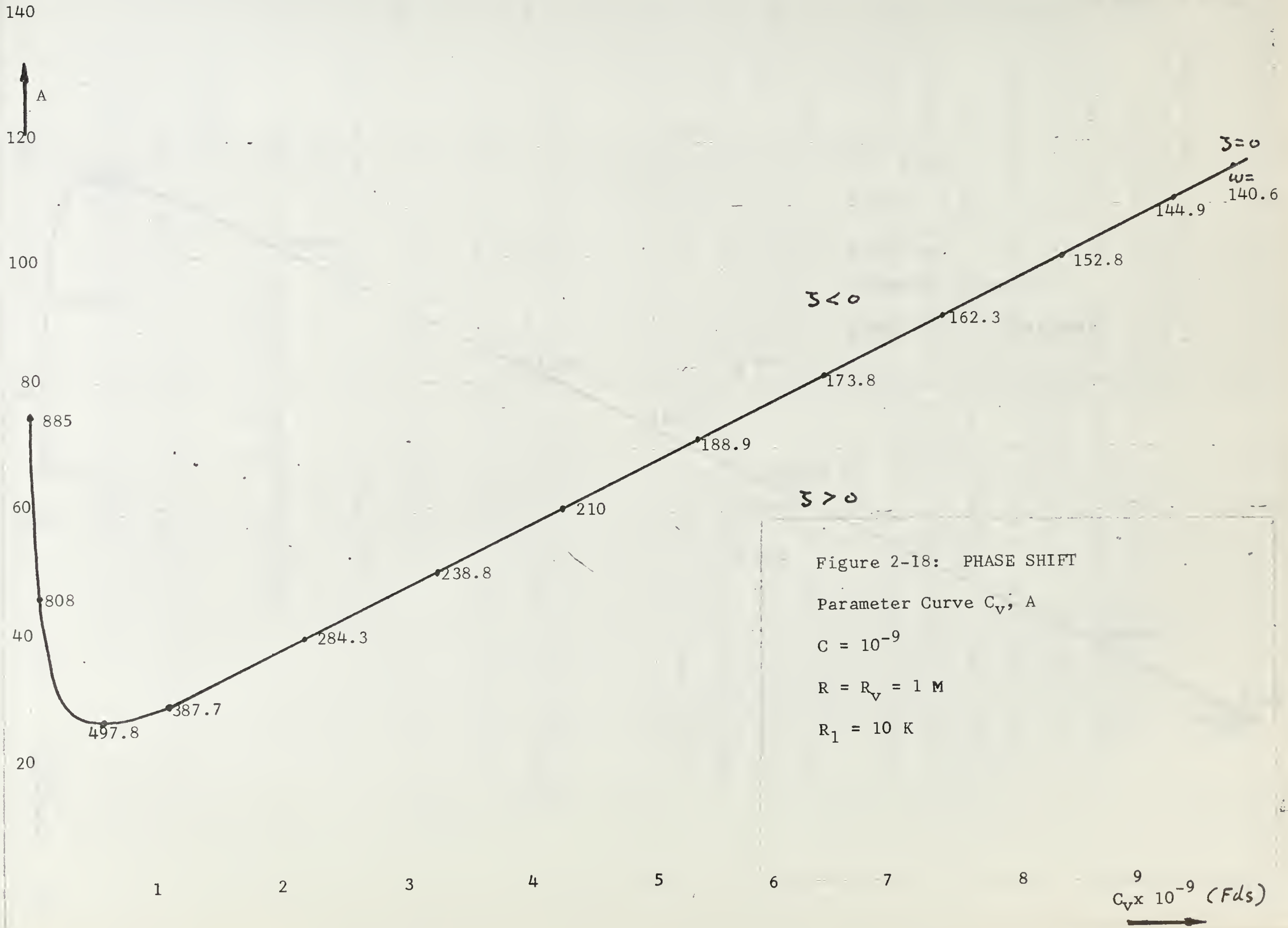


Figure 2-18: PHASE SHIFT

Parameter Curve C_v, A

$C = 10^{-9}$

$R = R_v = 1$ M

$R_1 = 10$ K

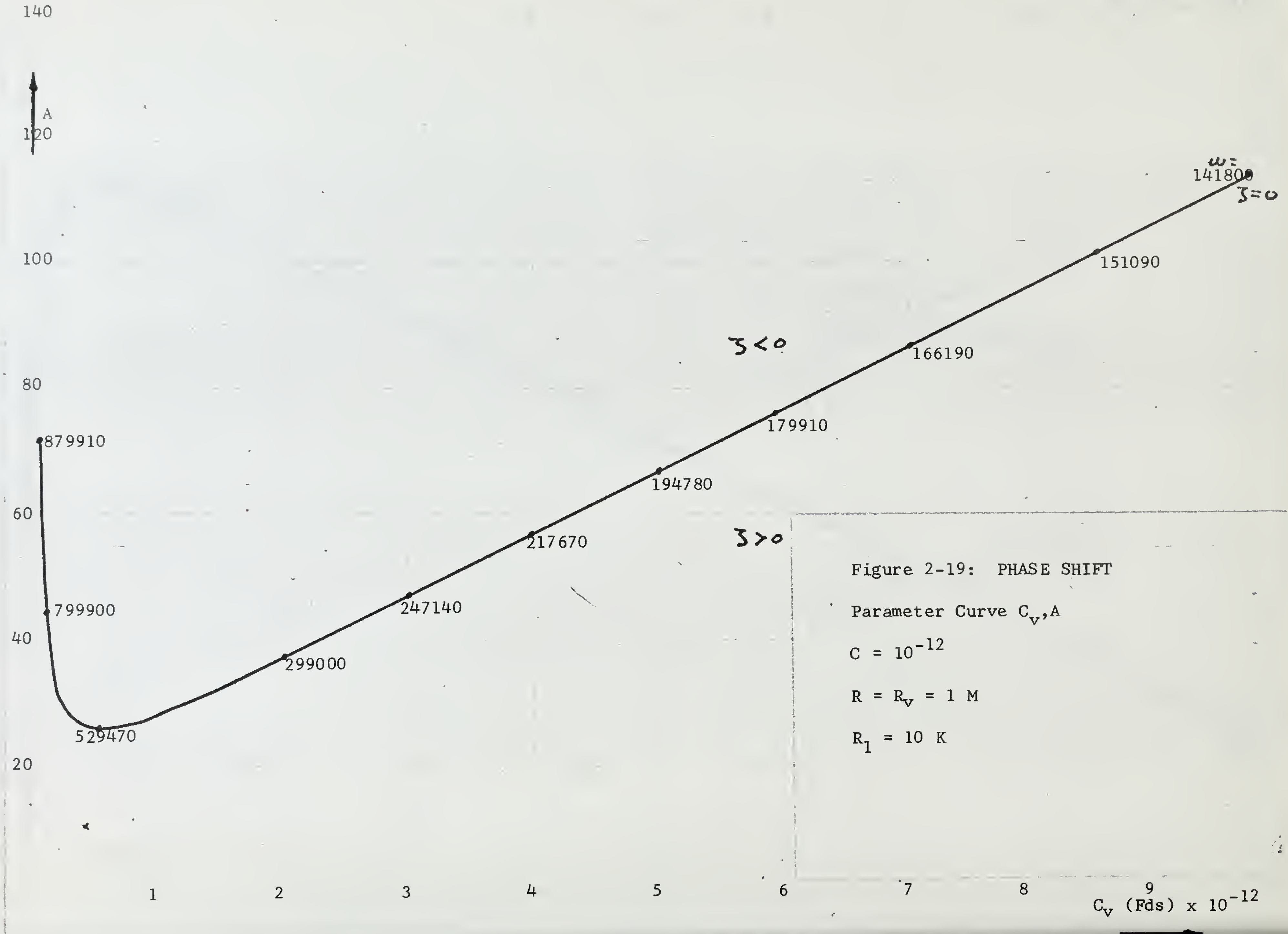


Figure 2-19: PHASE SHIFT

Parameter Curve C_v,A

$C = 10^{-12}$

$R = R_v = 1$ M

$R_1 = 10$ K

Results for Parameters C_v, A.

For various values of capacitors C we can have oscillations by varying C_v and A over a specific range of frequencies which is almost one decade. From Figure 2-18, frequency varies from ω = 140 to ω = 885 rad/sec while from Figure 2-19 ω = 141800 to ω = 879910 rad/sec.

The lower part of the band presents frequencies of linear variation of parameters C_v and A and give a perfect region of operation as far as sensitivity of oscillation is concerned. From Figure 2-18, the linear part is from ω = 140.6 to ω = 387.7 rad/sec.

There is a minimum value of gain so that the system will oscillate, namely $A \doteq 28$ for all values of C as it is clearly seen in Figures 2-17, 2-18, and 2-19. This result is in agreement with what has been already proven [1] analytically that for R R_1, $A \doteq 29$. For the same circuit, if $R_v = R$ and $C_v = C$ we find that

$$\frac{e}{E} = \frac{1}{1 - 5\left(\frac{X}{R}\right)^2 + \frac{R_1}{R}\left[3 - \left(\frac{X}{R}\right)^2\right] + j\left[\left(\frac{X}{R}\right)^3 - 6\frac{X}{R} - 4\left(\frac{R_1}{R}\right)\left(\frac{X}{R}\right)\right]}$$

In order to have oscillations, the imaginary part must be zero, or

$$\left(\frac{X}{R}\right)^3 - 6\frac{X}{R} - 4\left(\frac{R_1}{R}\right)\left(\frac{X}{R}\right) = 0$$

and if we set $X = \frac{1}{2\pi fC}$ and solve for frequency we get

$$f = \frac{1}{2\pi RC\left[6 + 4\frac{R_1}{R}\right]^{1/2}} \doteq \frac{1}{2\pi RC\sqrt{6}} \qquad (13)$$

for $R \gg R_1$.

Now the equation for the ratio $\frac{e}{E}$ using equation (7) and (8) we have $\frac{e}{E} = \frac{e}{-Ae_o}$ and for oscillations $\frac{e}{-Ae_o} = -\frac{1}{A}$.

So
$$-\frac{1}{A} = \frac{1}{1 - 5\left(\frac{X}{R}\right)^2 + \left(\frac{R_1}{R}\right)\left[3 - \left(\frac{X}{R}\right)^2\right]}$$

and solving for A we get

$$A = 29 + 23\,\frac{R_1}{R} + 4\left(\frac{R_1}{R}\right)^2$$

For $R \gg R_1$ as it is usually the case, $A \doteq 29$. But this is true for the special case where $C_v = C$ and $R_v = R$. We have proved now by curves of Figures 2-17, 2-18, and 2-19 that this holds true even if the outermost capacitor C_v is variable. Moreover it is shown from these figures that when A has its minimum value, C_v is approximately 0.5C. For example, for $C = 10^{-6}$ and for $A = 30$ $C_v = 0.5 \times 10^{-6}$, for $C = 10^{-12}$ $C_v = 0.5 \times 10^{-12}$.

The high part of the band is almost vertical as in Figure 2-18 from $\omega = 699$ to $\omega = 885$ rad/sec. If we consider that A is constant, we must have means of making very small adjustments of C_v in order to have oscillations at a specific frequency.

Near the middle of the band where $A \doteq 30$ we may have considerable variations in frequency for only slight change in gain. So if the tube is subject to gain variations this is not a recommended area of operation.

2-5. Parameter plane curves C_v versus R_1 for different ranges of frequencies and different gains.

Rearrange equation (9) so that we will have as parameters R_1 and C_v.

$$C^2\left[C_vR_1\,(2RR_v + R^2) + C_v\,(AR_vR^2 + R_vR^2)\right]S^3$$

$$+\left[C_vR_1\,(R_vC + RC) + C_v\,(3RR_vC + 2R^2C)\right.$$

$$\left. + R_1(2RC^2) + R^2C^2\right]S^2 + \left[C_v(R_v + R) + R_1(C) + 3RC\right]S + 1 = 0$$

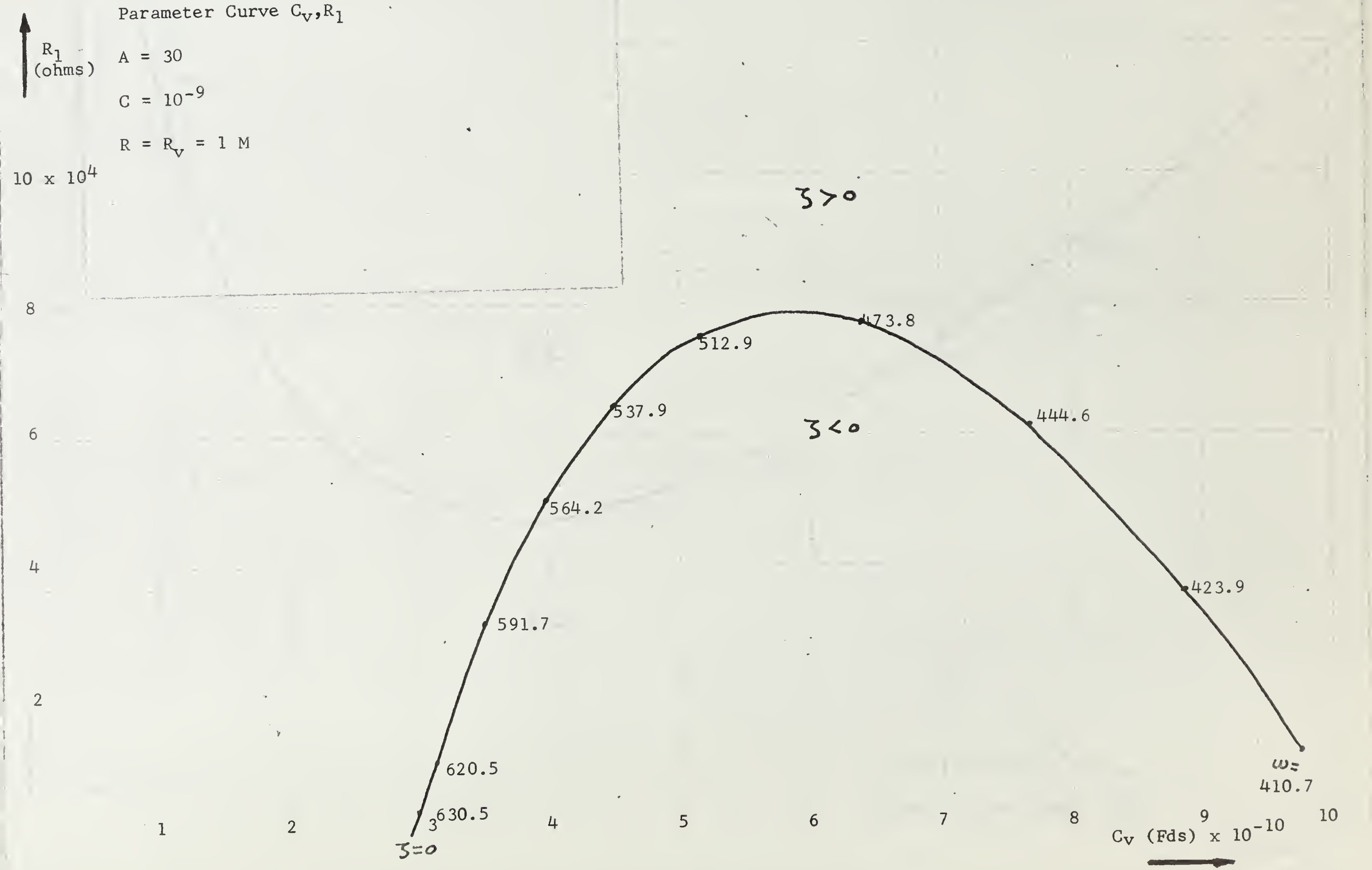
Figure 2-20: PHASE SHIFT

Parameter Curve C_v, R_1

$A = 30$

$C = 10^{-9}$

$R = R_v = 1$ M



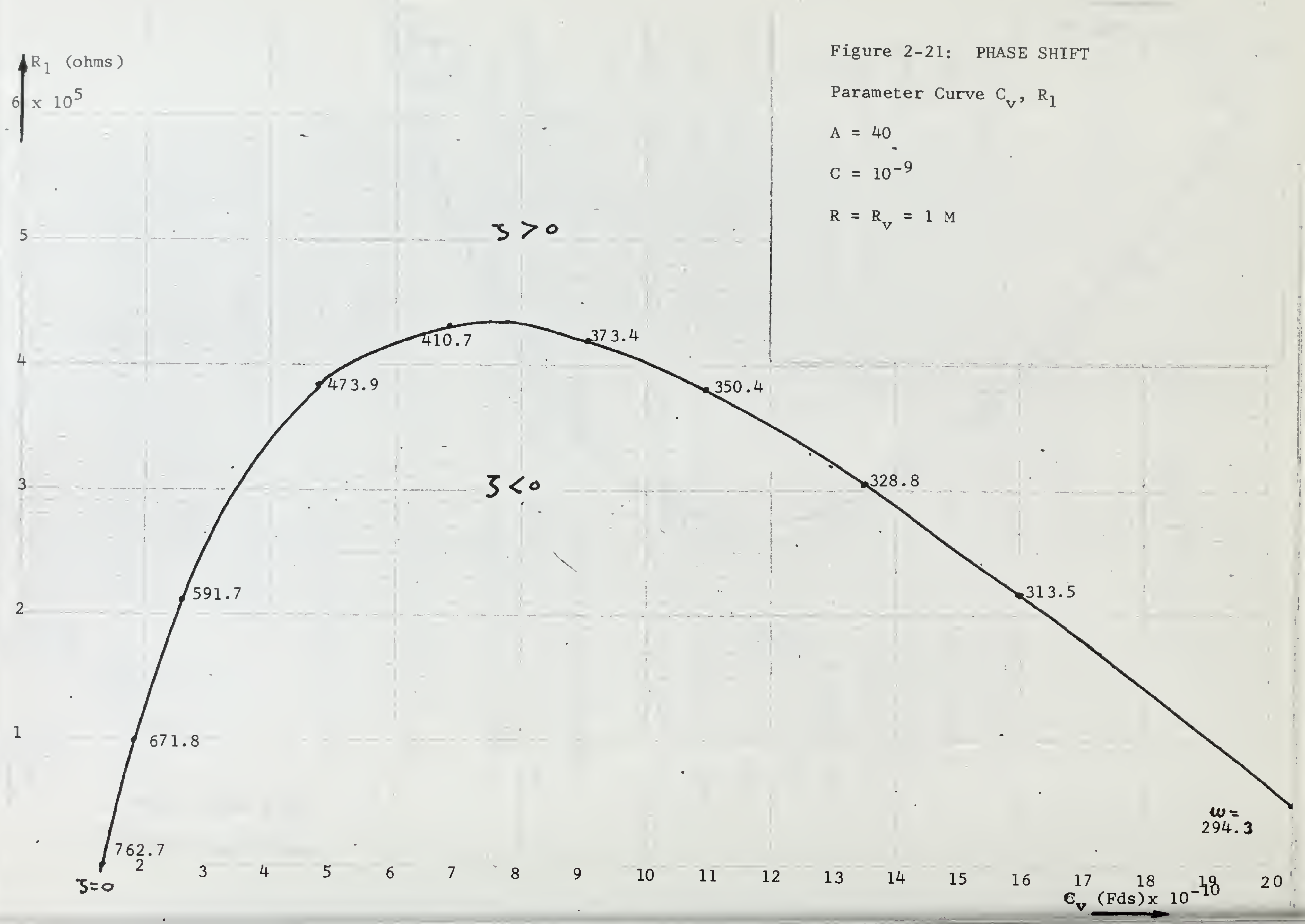


Figure 2-21: PHASE SHIFT

Parameter Curve C_v, R_1

$A = 40$

$C = 10^{-9}$

$R = R_v = 1$ M

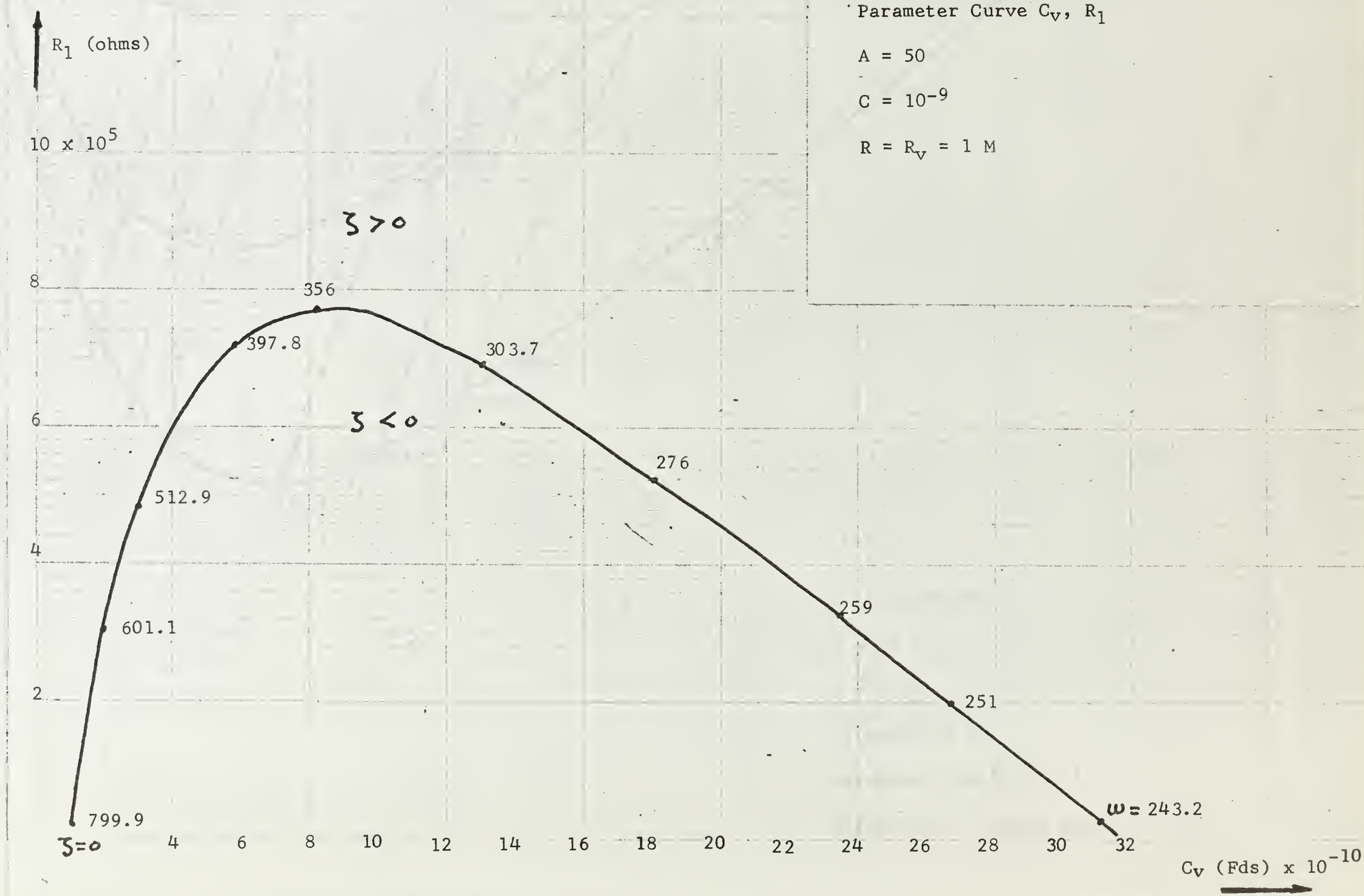


Figure 2-22: PHASE SHIFT

Parameter Curve C_v, R_1

$A = 50$

$C = 10^{-9}$

$R = R_v = 1$ M

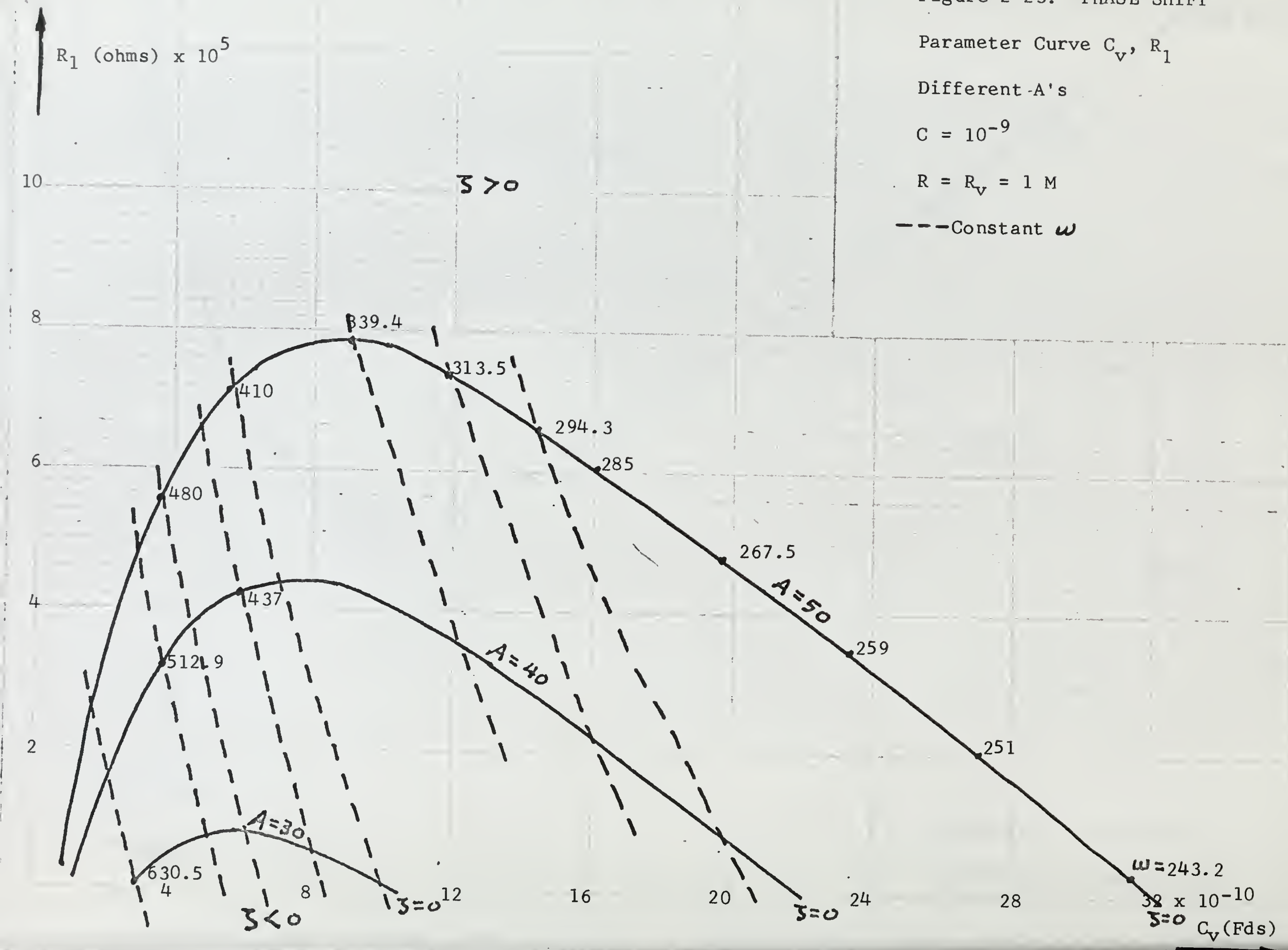


Figure 2-23: PHASE SHIFT

Parameter Curve C_v, R_1

Different A's

$C = 10^{-9}$

$R = R_v = 1$ M

- - - Constant ω

Give again typical characteristic values as follows:

$$R = R_v = 1\ M$$

$$A = 20,\ 30,\ 40,\ 50$$

$$C = 10^{-9}$$

For A = 20

$$\left[C_vR_1(0.3 \times 10^{-5}) + C_v(0.201 \times 10^2)\right] S^3 + \left[C_vR_1(0.2 \times 10^{-2})\right.$$

$$\left. + C_v(0.5 \times 10^4) + R_1(0.2 \times 10^{-11}) + 0.1 \times 10^{-5}\right] S^2$$

$$+\left[C_v(0.2 \times 10^7) + R_1(0.1 \times 10^{-8}) + 0.3 \times 10^{-2}\right] S + 1 = 0$$

For A = 30

$$\left[C_vR_1(0.3 \times 10^{-5}) + C_v(0.301 \times 10^2)\right] S^3 + \text{(as above)}$$

For A = 40

$$\left[C_vR_1(0.3 \times 10^{-5}) + C_v(0.401 \times 10^2)\right] S^3 + \ldots.$$

For A = 50

$$\left[C_vR_1(0.3 \times 10^{-5}) + C_v(0.501 \times 10^2)\right] S^3 + \ldots.$$

Higher Frequencies

To examine the situation for higher frequencies we set the capacitors at a value $C = 10^{-12}$ fd and the other parameters the same.

So equation (9) becomes:

For A = 30

$$\left[C_vR_1(0.3 \times 10^{-11}) + C_v(0.31 \times 10^{-4})\right] S^3 + \left[C_vR_1(0.2 \times 10^{-5})\right.$$

$$\left. + C_v(0.5 \times 10^1) + R_1(0.2 \times 10^{-17}) + 0.1 \times 10^{-11}\right] S^2$$

$$+\left[C_v(0.2 \times 10^7) + R_1(0.1 \times 10^{-11}) + 0.3 \times 10^{-5}\right] S + 1 = 0$$

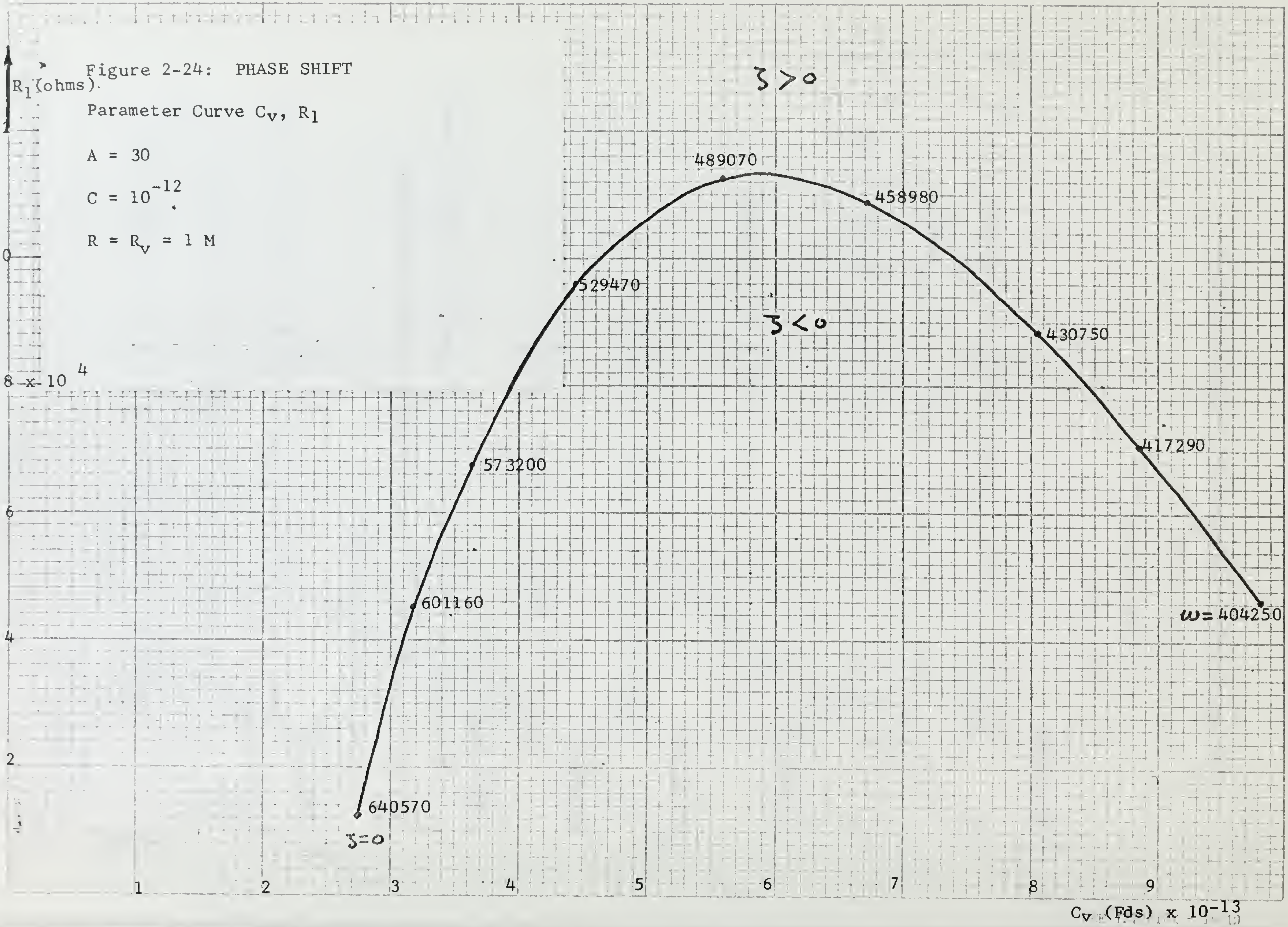


Figure 2-24: PHASE SHIFT

Parameter Curve C_v, R_1

$A = 30$

$C = 10^{-12}$

$R = R_v = 1$ M

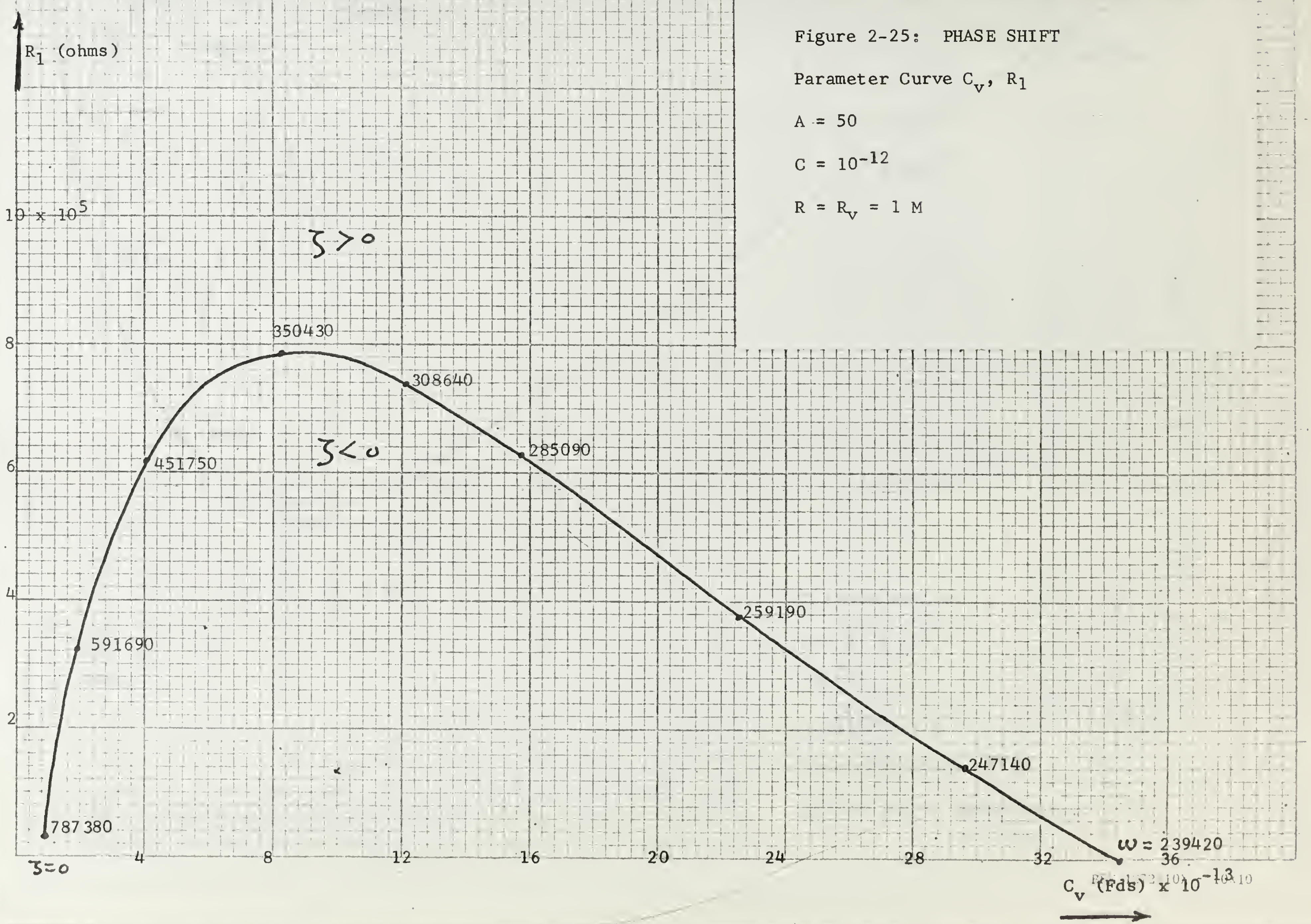


Figure 2-25: PHASE SHIFT

Parameter Curve C_v, R_1

$A = 50$

$C = 10^{-12}$

$R = R_v = 1\ M$

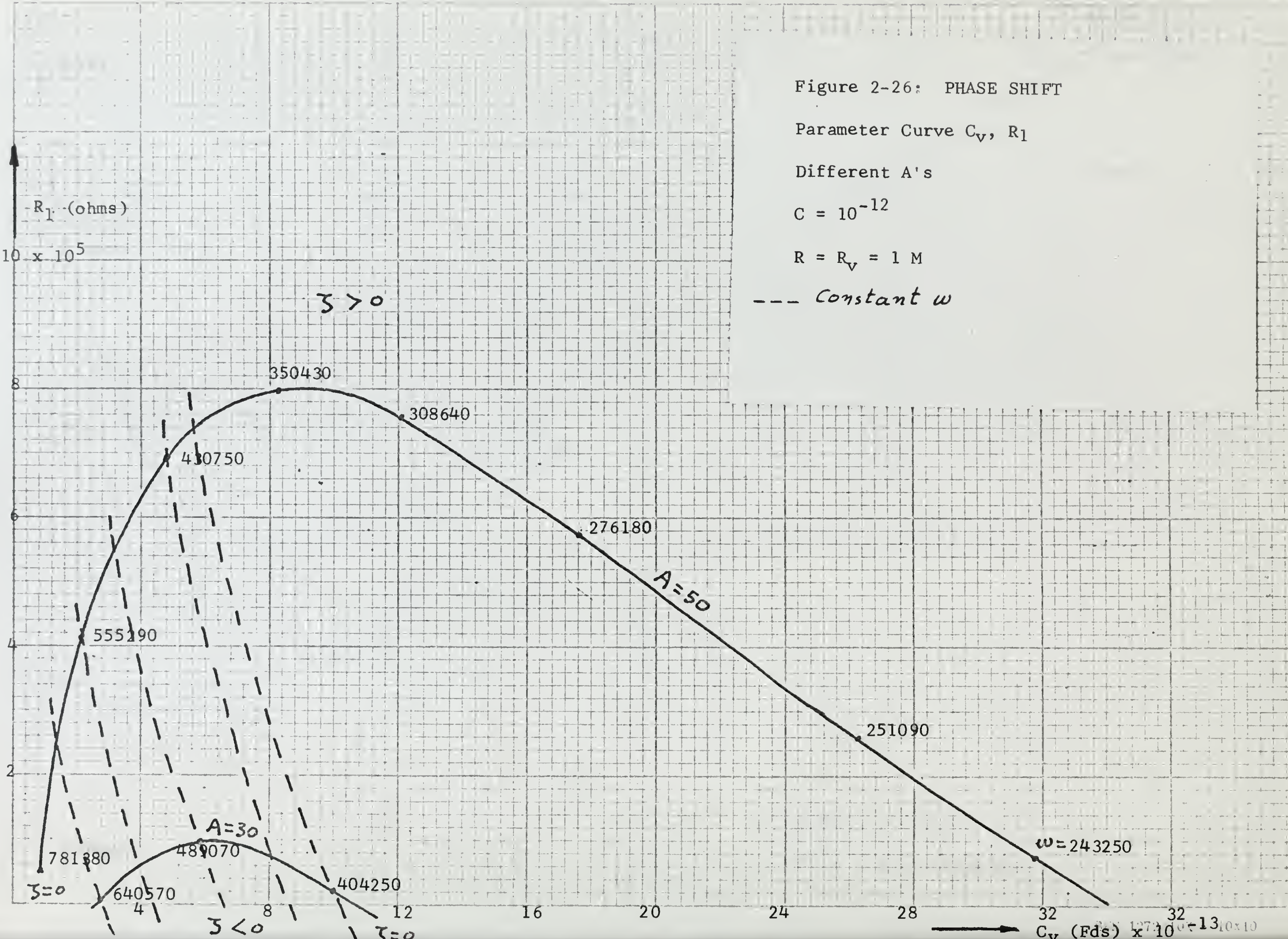


Figure 2-26: PHASE SHIFT

Parameter Curve C_v, R_1

Different A's

$C = 10^{-12}$

$R = R_v = 1$ M

--- Constant ω

For A = 50

$$\left[C_vR_1(0.3 \times 10^{-11}) + C_v(0.51 \times 10^{-4})\right] S^3$$

$$+\left[C_vR_1(0.2 \times 10^{-5}) + C_v(0.5 \times 10^{1}) + R_1(0.2 \times 10^{-17})\right.$$

$$\left.+ 0.1 \times 10^{-11}\right] S^2 + \left[C_v(0.2 \times 10^{7}) + R_1(0.1 \times 10^{-11})\right.$$

$$\left.+ 0.3 \times 10^{-5}\right] S + 1 = 0.$$

Results for Parameters R_1, C_v

For gain A = 20 there are no real values at R_1, C_v that can produce oscillations which agree with previous results, that minimum gain is about 28.

The $\omega = 0$ curves have a parabolic shape with steeper slope at higher frequencies. For example, in Figure 2-22 for frequencies $\omega = 397.8$ to $\omega = 799.9$ rad/sec, the slope is steeper positive than from $\omega = 323.7$ rad/sec (negative slope).

There is a maximum value of R_1 for a given gain as for A = 30 maximum R_1 = 80 K and this value is higher for higher gains, as for A = 50 maximum R_1 = 780 K .

The range of values of C_v is larger for higher gains. For A = 30 C_v varies from 2.9×10^{-10} to 9.8×10^{-10} farads while for A = 50 C_v = 1.3×10^{-10} to 20.5×10^{-10} farads.

The range over which the frequency can be controlled by R_1, C_v is wider for higher gains. For A = 40, ω varies between 294.3 to 762.7 rad/sec and for A = 50 between 243.3 to 799.9 rad/sec.

Change of $R_1 = \dfrac{R_1^1 R_p}{R_1^1 + R_p}$ can be accomplished either intentionally by changing the load resistance $R_1^{\,1}$ or by eventual changing of tube plate resistance R_p due to temperature or supply voltage variations. From the

$\zeta = 0$ curves it is evident that we have better frequency stability due to small R_1 changes at low frequencies rather than high frequencies because the rate of frequency change is less at low frequencies.

This fact has been proven analytically as follows: We define:

$$\text{frequency sensitivity} \equiv \frac{1}{\text{freq. stability}}$$

also

$$\text{frequency stability} = \frac{\text{change of oscil. freq.}}{\text{oscillation frequency}}$$

Since we are interested in variation of parameter R_1, we can associate df with dR_1 as follows:

$$\frac{df}{f} = K_1 \frac{dR_1}{R_1} \tag{15}$$

from equation (13)

$$f = \frac{1}{2\pi RC \left[6 + 4 \frac{R_1}{R}\right]^{1/2}}$$

and differentiating we get:

$$\frac{df}{dR_1} = \frac{1}{2\pi RC} \cdot \frac{-2}{R(6 + 4 \frac{R_1}{R})^{3/2}}$$

or

$$\frac{df}{f} = -\frac{R_1}{2R_1 + 3R} \cdot \frac{dR_1}{R_1} = K_1 \frac{dR_1}{R_1} \tag{16}$$

where

$$K_1 = -\frac{R_1}{2R_1 + 3R}$$

But $R_1 = \dfrac{R_1^1 R_p}{R_1^1 + R_p}$ and differentiating this with respect to R_p and plunging into equation (16) we get

$$\frac{df}{f} = \frac{R_1^{1\,2}}{2R_1^1 + 3R(1 + \frac{R_1^{\,1}}{R_p}) \quad R_1^1 + R_p} \quad \frac{dR_p}{R_p}$$

and calling the first factor of the right hand part of the equation K_2, we get

$$(17) \qquad \frac{df}{f} = K_2 \frac{dR_p}{R_p}$$

Figure 2-23 gives the family of curves for different gains A where the constant ω curves have been superimposed. Those constant ω curves appear to have almost the same slope.

Figures 2-24, 2-25, 2-26 are drawn for higher operating frequencies and it is shown that frequency variations give the same pattern.

2-6. Sensitivity curves ω versus C_v with R_v varying.

The sensitivity in frequency of the oscillator can be studied better with respect to one parameter only, and then with respect to the other.

So the next curves have been drawn as frequency versus one parameter, while we do not take into account the second parameter of the parameter plane. Nevertheless the other parameter is varying, as stated on each of the curves.

The pattern of frequency variation versus C_v is the same for different values of C as shown in Figures 2-27, 2-28, and 2-29. For lower frequencies one needs large variations of C_v to produce a relative large change in frequency. For example, from Figure 2-28, a change of C_v from 1.9×10^{-9} to 3.30×10^{-9} fd changes frequency from $.17 \times 10^3$ rad/sec to $.225 \times 10^3$ rad/sec for a gain A = 50. As the gain goes lower, a lesser and lesser variation of C_v is needed to produce the same change in frequency.

There is a point on each curve (peak of curves) which produces in-

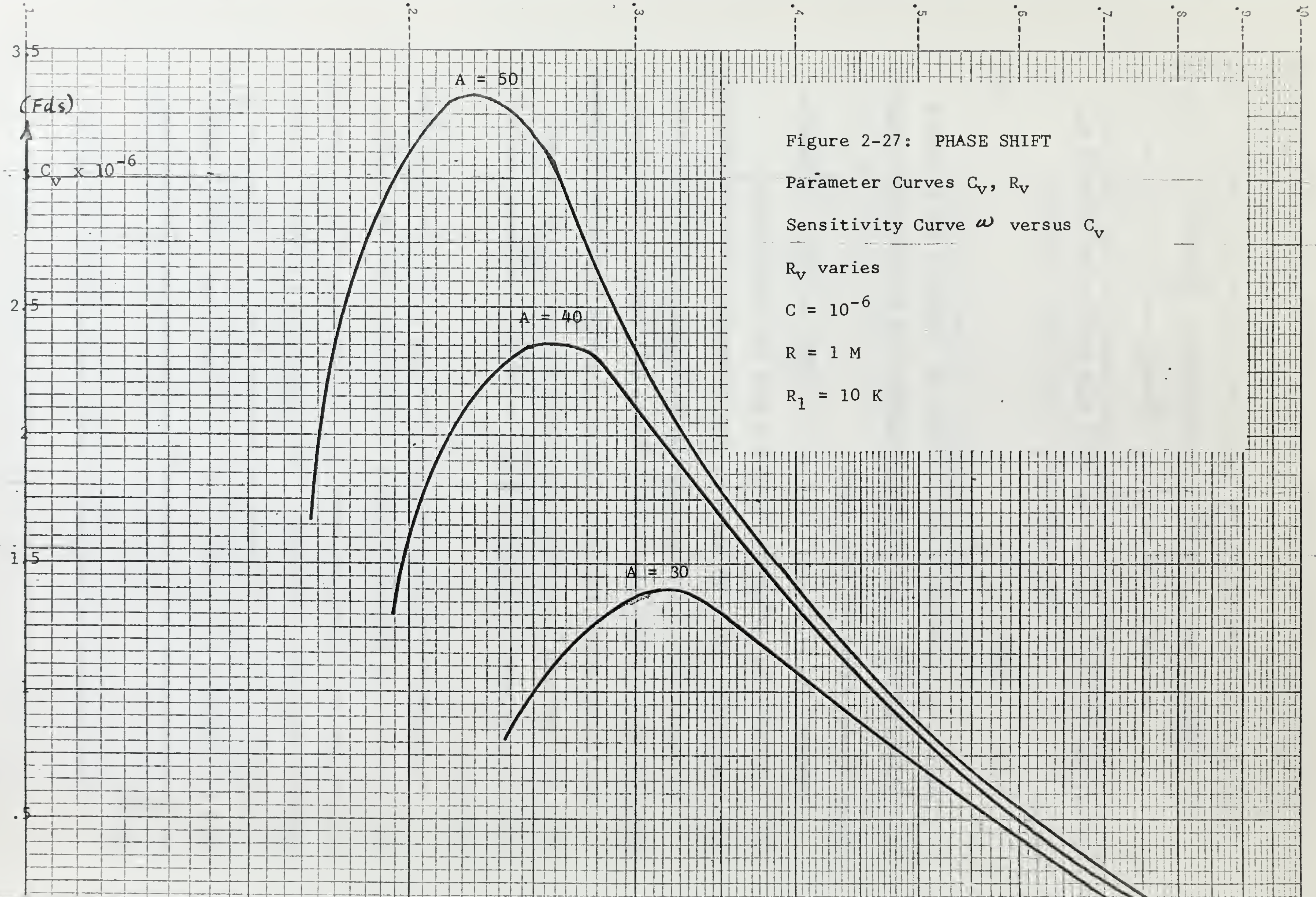


Figure 2-27: PHASE SHIFT

Parameter Curves C_v, R_v

Sensitivity Curve ω versus C_v

R_v varies

$C = 10^{-6}$

$R = 1$ M

$R_1 = 10$ K

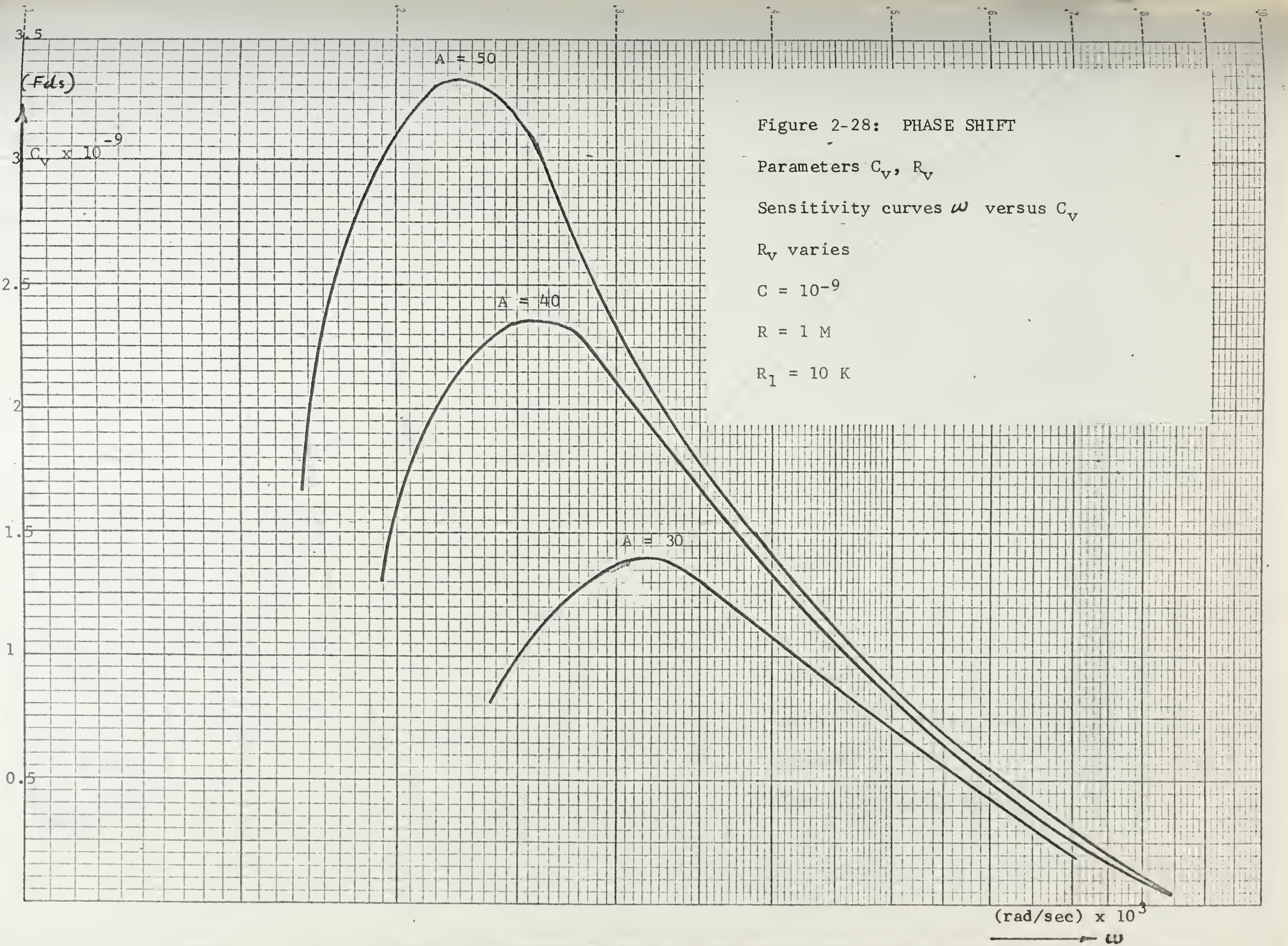


Figure 2-28: PHASE SHIFT

Parameters C_v, R_v

Sensitivity curves ω versus C_v

R_v varies

$C = 10^{-9}$

$R = 1$ M

$R_1 = 10$ K

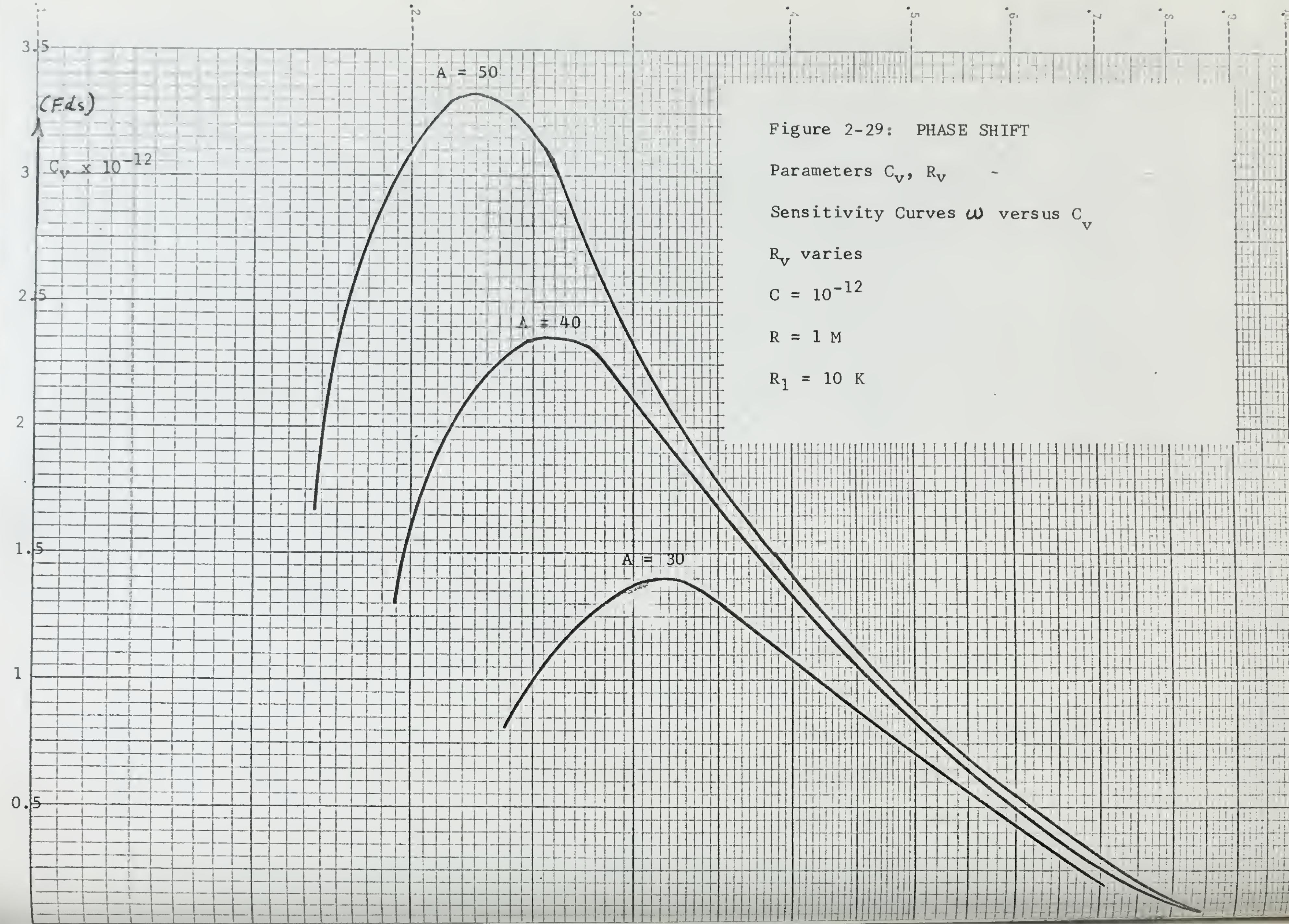


Figure 2-29: PHASE SHIFT

Parameters C_v, R_v -

Sensitivity Curves ω versus C_v

R_v varies

$C = 10^{-12}$

$R = 1$ M

$R_1 = 10$ K

stability because the slightest change of C_V would cause considerable change in frequency. As we go lower in gain the peak point flattens and and shifted to higher frequencies so the same amount of change in C_V causes a severe change in frequency.

For higher frequencies ($\omega > .225 \times 10^3$ rad/sec for A = 50, Figure 2-28), as we decrease C_V, frequency is increasing in logarithmic fashion and the lower the values of C_V, the higher the frequency. For example, a change of C_V from 3 μf to 2.5 μf changes frequency from 0.265 Krad/sec to .295 Krad/sec while a change of C_V from 2 μf to 1.5 μf changes frequency from .335 Kcps to .395 Kcps (Figure 2-28, A = 50).

2-7. Sensitivity curves ω versus R_V with C_V varying.

The pattern of frequency variation is the same for different values of C which give different ranges of frequencies (Figures 2-30, 2-31, and 2-32)

From Figure 2-32. For low frequencies about ω = .25 Mrad/sec variation of R_V gives the same variation in frequency for gains close to A = 50. But as the gain is lower the change in frequency is higher. For a change of R_V from 3 M to 2 M there is a change in frequency from .185 Mrad/sec to .197 Mrad/sec for A = 50 and from .263 to .303 for A = 30.

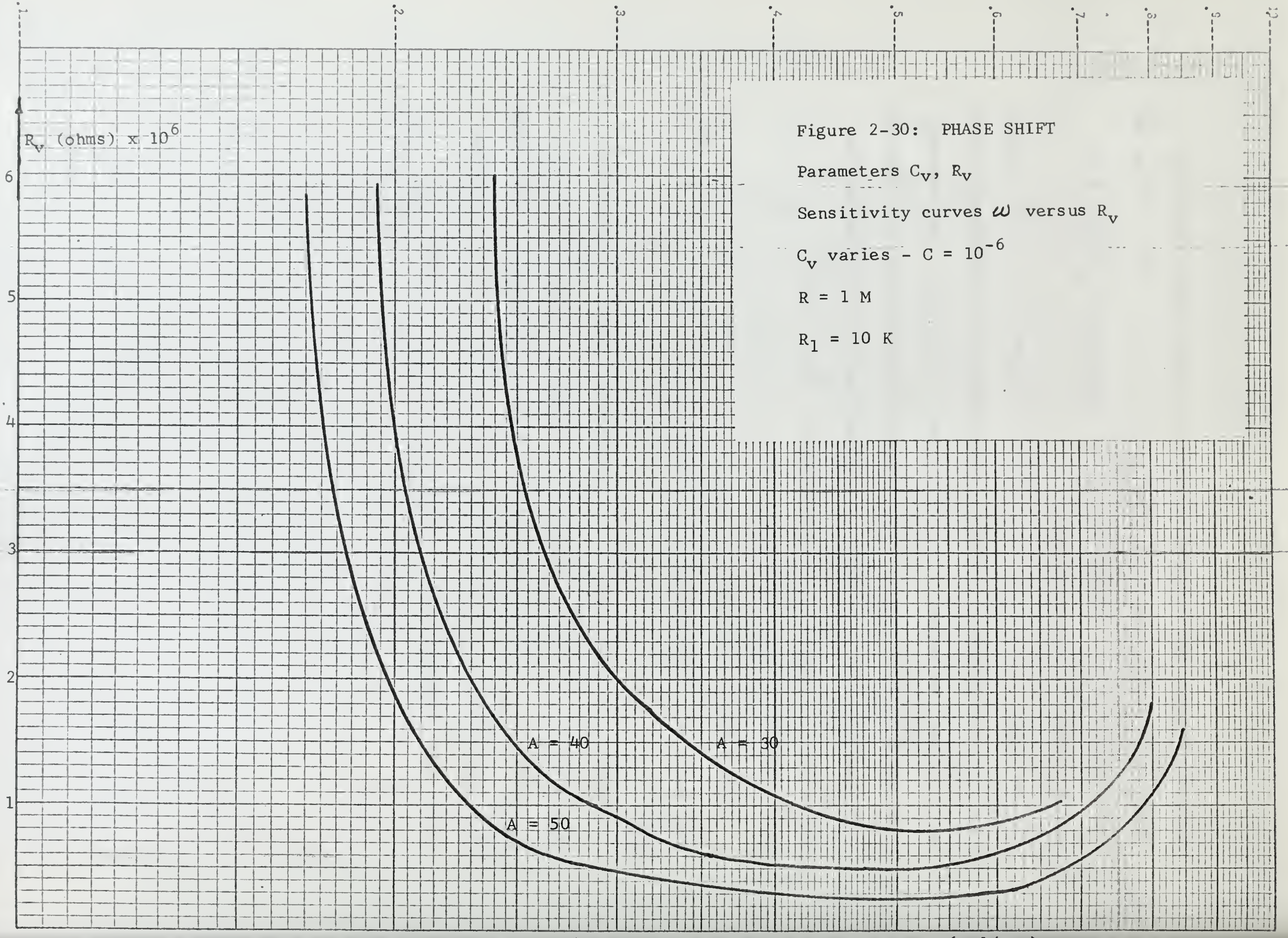


Figure 2-30: PHASE SHIFT

Parameters C_v, R_v

Sensitivity curves ω versus R_v

C_v varies - $C = 10^{-6}$

R = 1 M

R_1 = 10 K

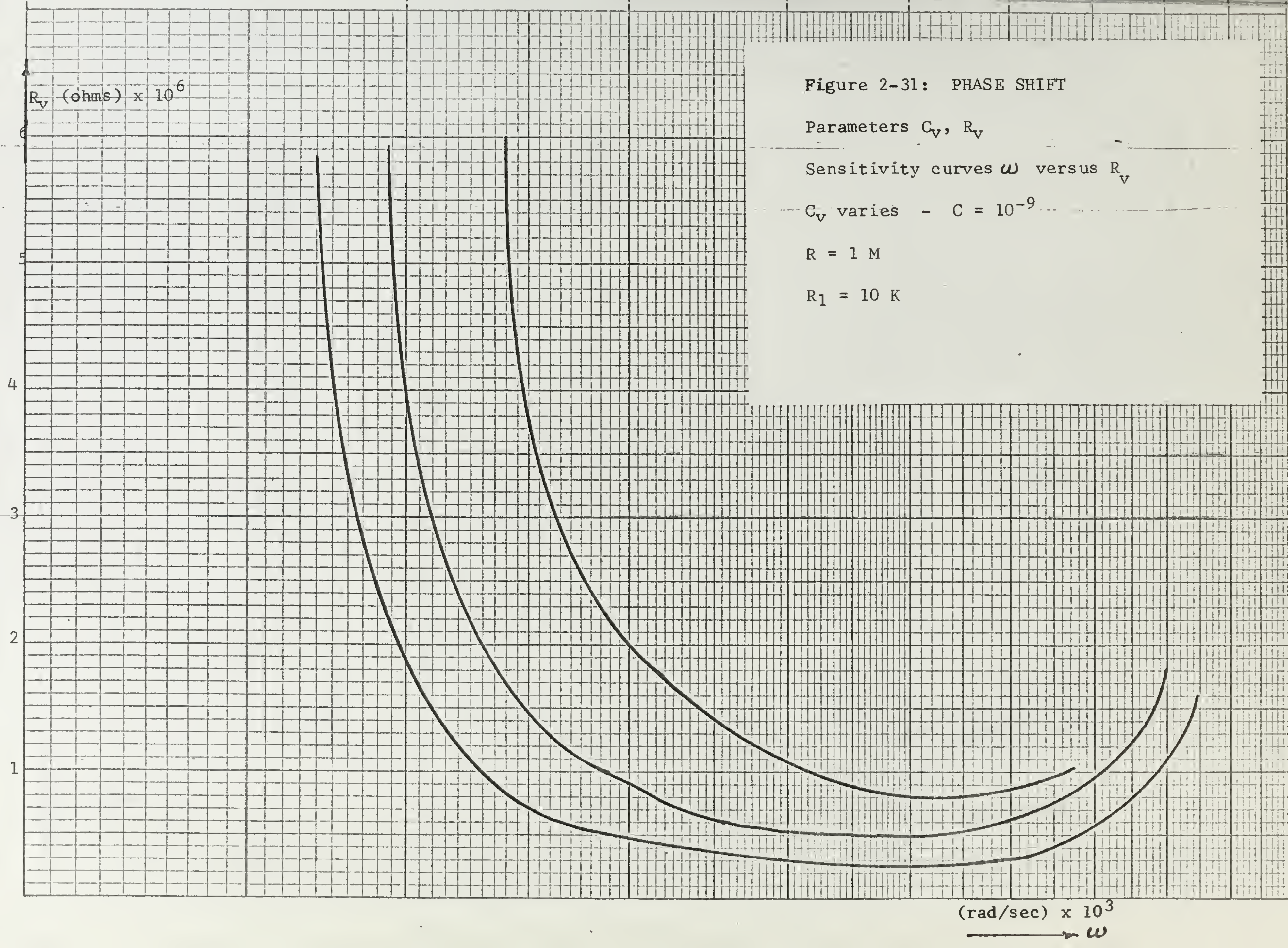


Figure 2-31: PHASE SHIFT

Parameters C_v, R_v

Sensitivity curves ω versus R_v

C_v varies - $C = 10^{-9}$

$R = 1$ M

$R_1 = 10$ K

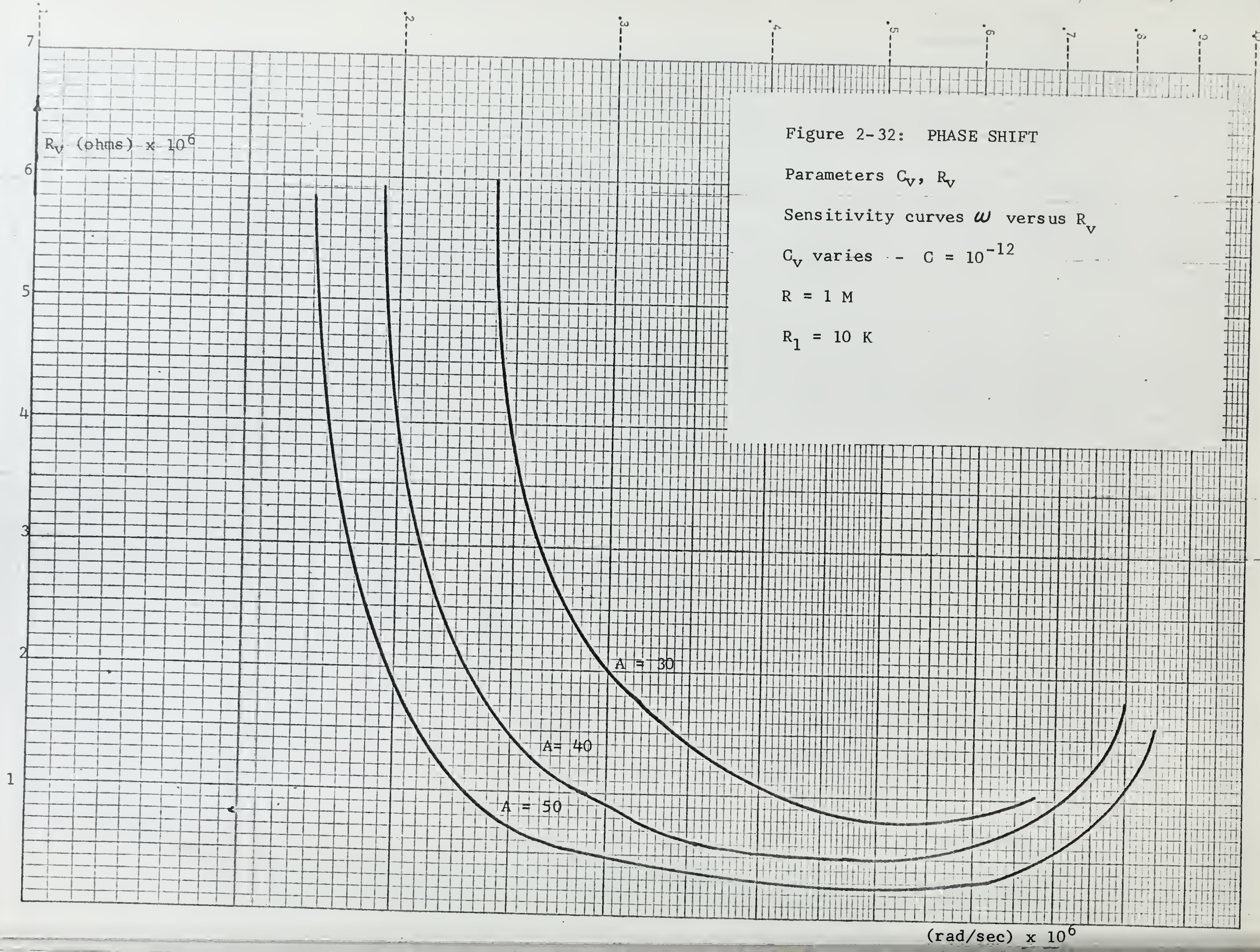


Figure 2-32: PHASE SHIFT

Parameters C_v, R_v

Sensitivity curves ω versus R_v

C_v varies - $C = 10^{-12}$

R = 1 M

R_1 = 10 K

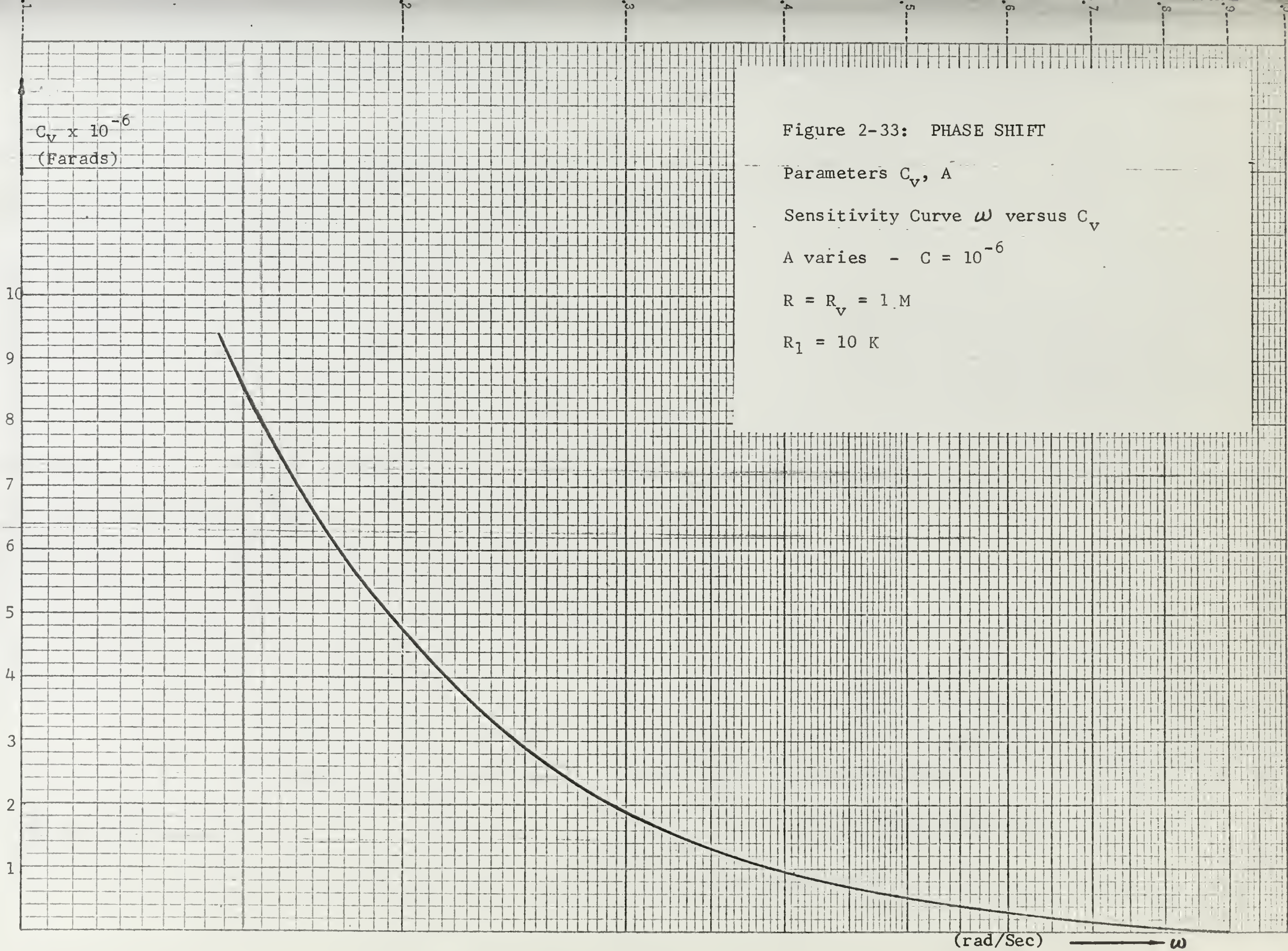


Figure 2-33: PHASE SHIFT

Parameters C_v, A

Sensitivity Curve ω versus C_v

A varies - $C = 10^{-6}$

$R = R_v = 1$ M

$R_1 = 10$ K

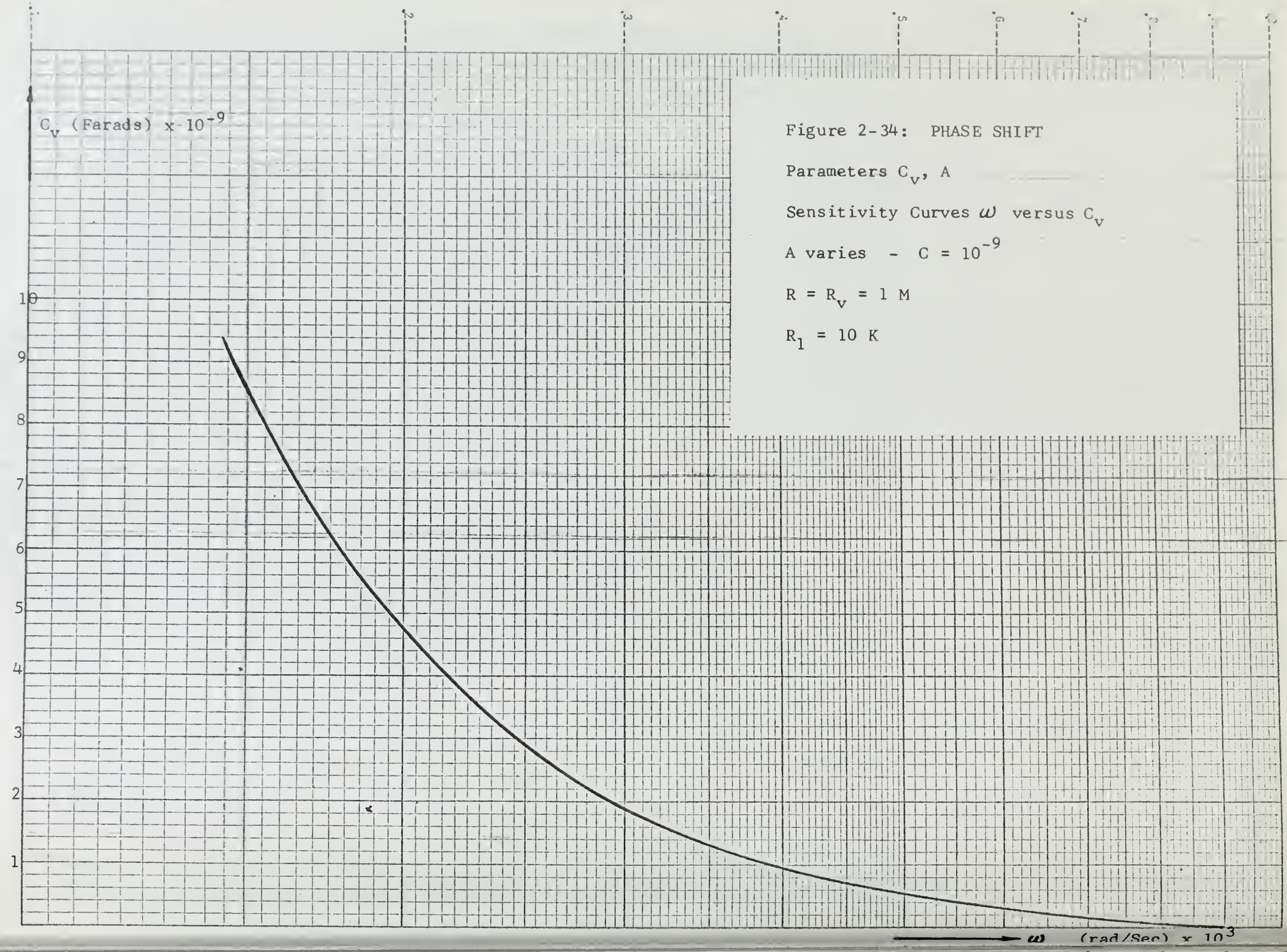


Figure 2-34: PHASE SHIFT

Parameters C_v, A

Sensitivity Curves ω versus C_v

A varies - $C = 10^{-9}$

$R = R_v = 1$ M

$R_1 = 10$ K

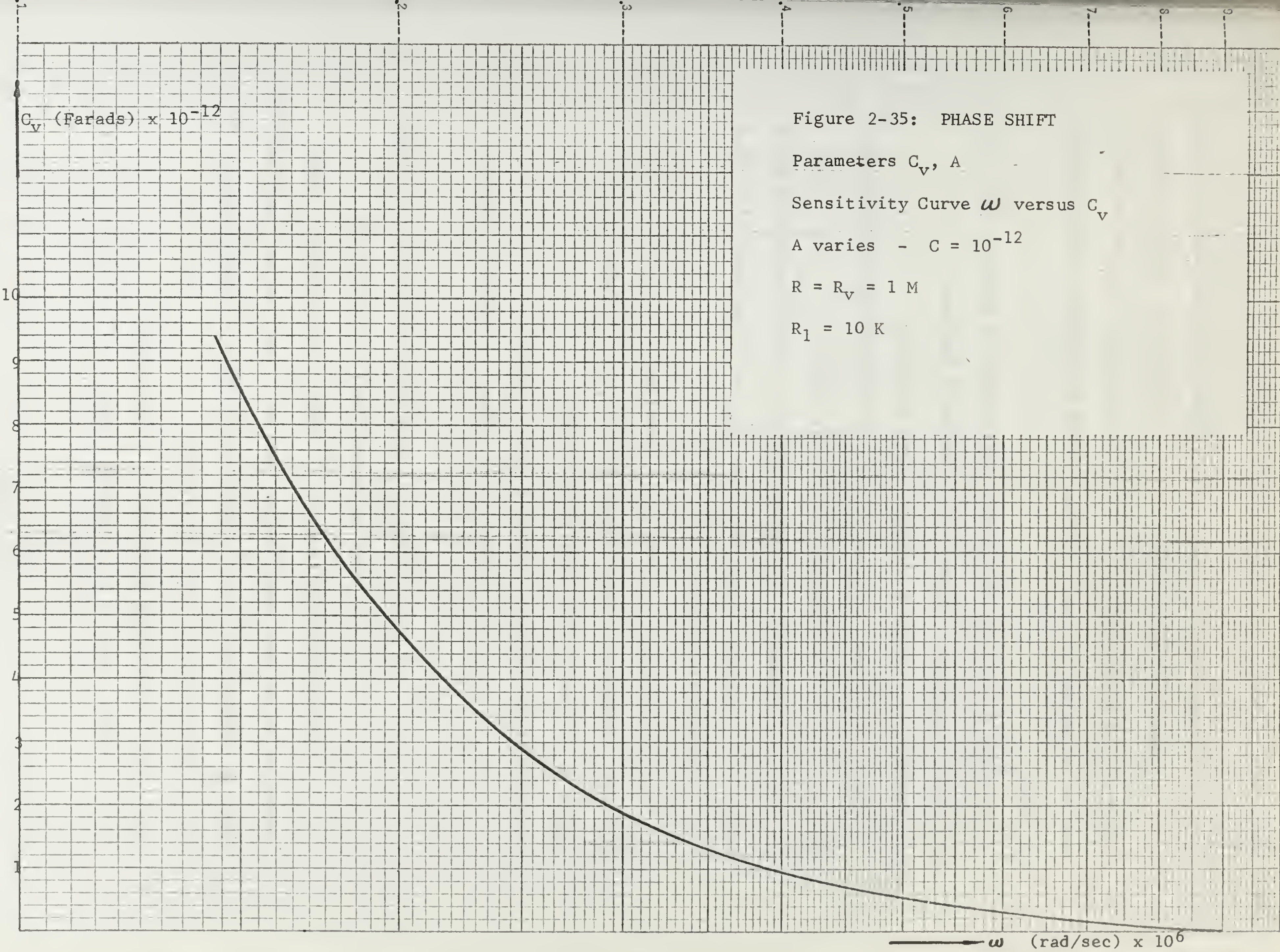


Figure 2-35: PHASE SHIFT

Parameters C_v, A

Sensitivity Curve ω versus C_v

A varies - $C = 10^{-12}$

$R = R_v = 1$ M

$R_1 = 10$ K

In the middle range of frequencies (Figure 2-32) the higher the gain is, the flatter the curve, so there is an instability in frequency which is more severe for higher gains. In other words small variations of R_V cause extensive variations of frequency when we operate on this part of the curve.

For high frequencies the curves present again a workable area, but here small changes of R_V cause large changes of frequency.

2-8. Sensitivity curves ω versus C_V with A varying.

The curves present the same pattern of variation in frequency for different values of C (Figures, 2-23, 2-24, 2-25).

For low frequencies, the curves present a good workable part where variation of C_V causes a reasonable variation of frequencies (for about .3 krad/sec)in graph 39.

For high frequencies a very small variation of C_V causes a large variation in frequency as for change of C_V 1 μf to 0.5 μf, frequency changes from .5 to .5 krad/sec (Figure 2-34).

2-9. Sensitivity curves ω versus A with C_V varying.

The curves are the same for different values of C_V (Figures 2-36, 2-37, 2-38).

There is a unique minimum in gain about A = 28 for all values of C. For gain lower than that we cannot sustain oscillations.

For gains from 28 to about 70 there are two points on each curve where we can have oscillations, one at low and one at high frequencies, for example, for A = 60 (Figure 2-38), we can have oscillations at ω = .21 Mrad/sec or at ω = .86 Mrad/sec.

2-10. Sensitivity curves ω versus R_1 with C_V varying.

As the value of C is lowered from $C = 10^{-9}$ fd to $C = 10^{-12}$ fd,

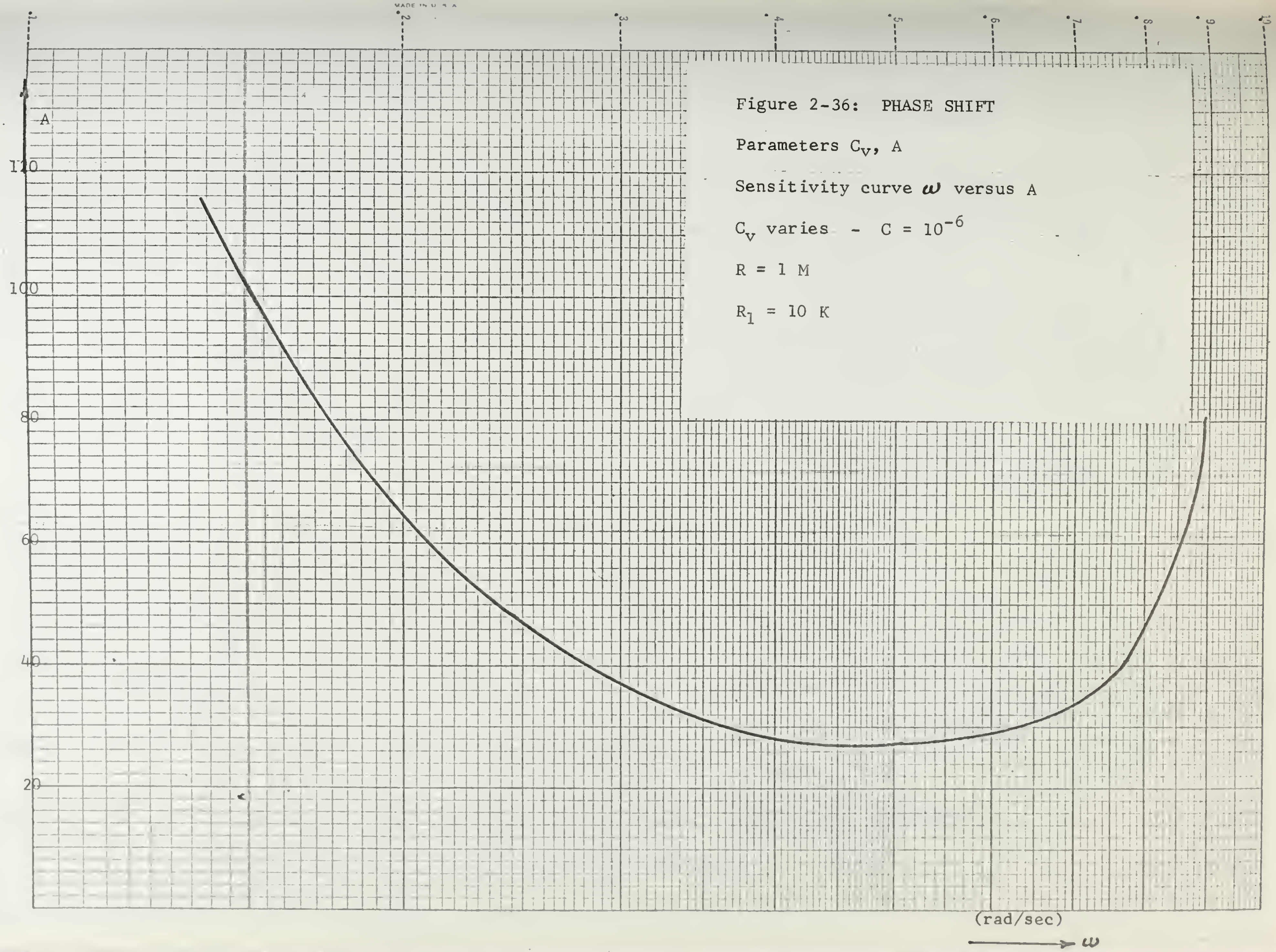


Figure 2-36: PHASE SHIFT

Parameters C_v, A

Sensitivity curve ω versus A

C_v varies - $C = 10^{-6}$

R = 1 M

R_1 = 10 K

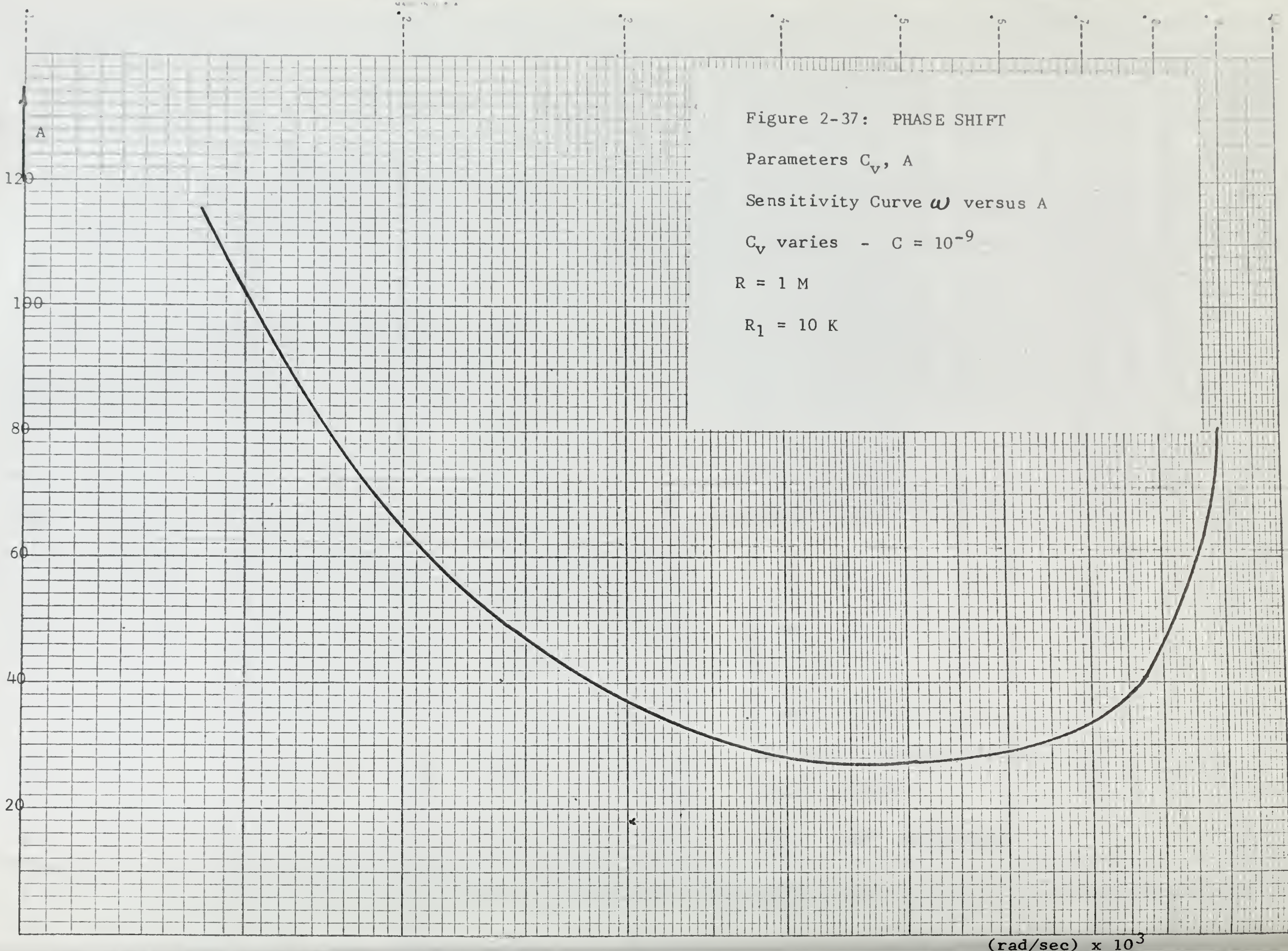


Figure 2-37: PHASE SHIFT

Parameters C_v, A

Sensitivity Curve ω versus A

C_v varies - $C = 10^{-9}$

R = 1 M

R_1 = 10 K

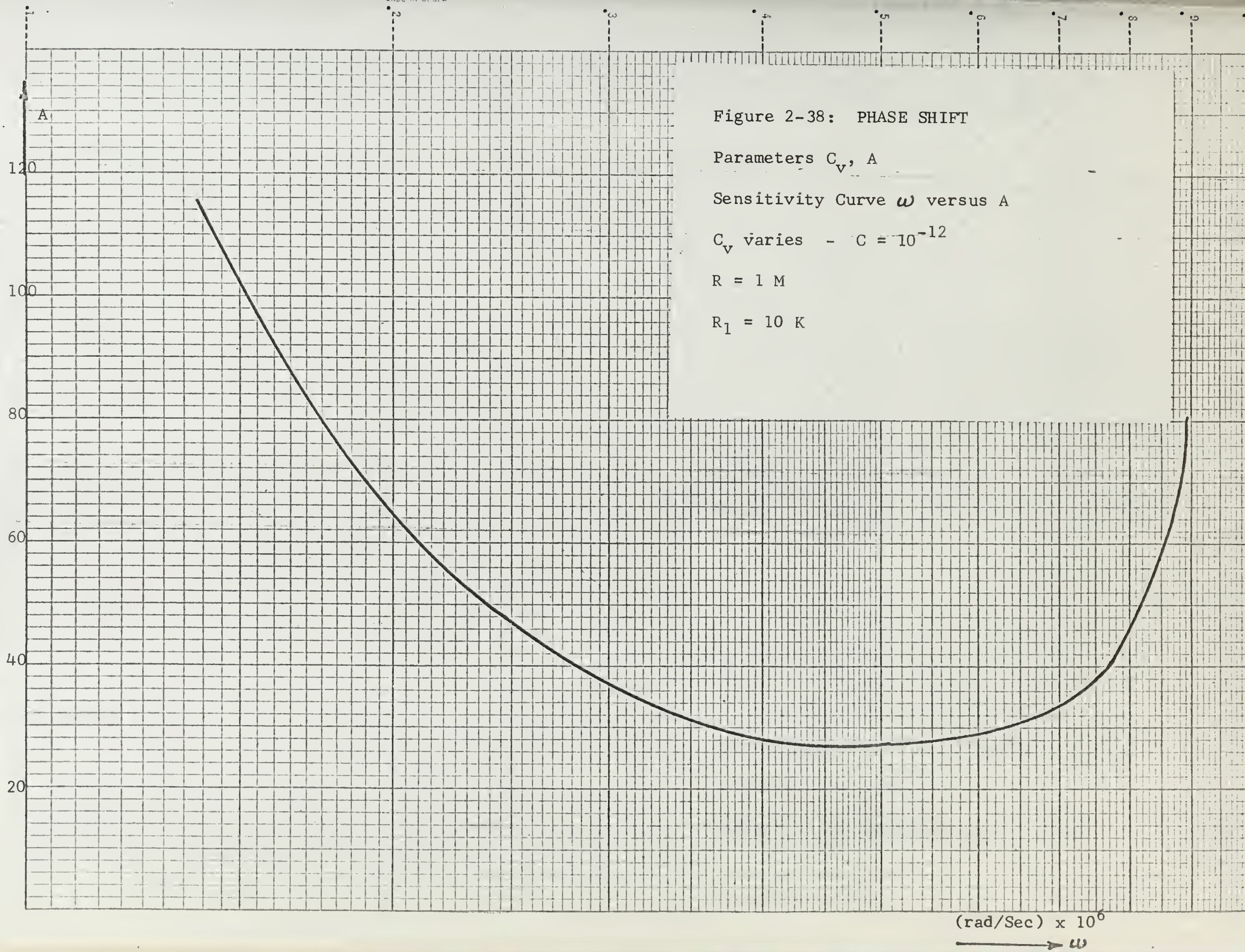


Figure 2-38: PHASE SHIFT

Parameters C_v, A

Sensitivity Curve ω versus A

C_v varies - $C = 10^{-12}$

$R = 1\ M$

$R_1 = 10\ K$

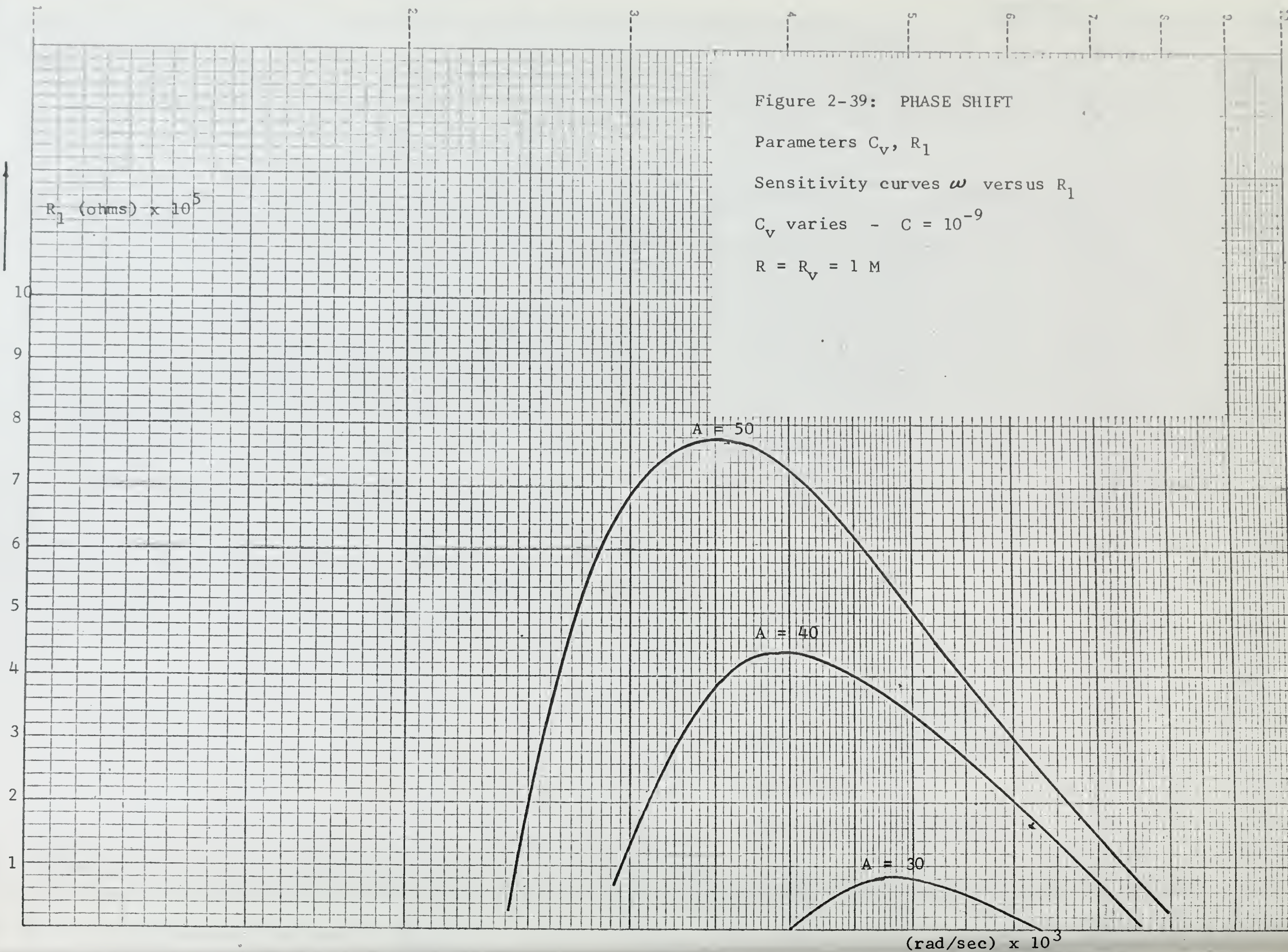


Figure 2-39: PHASE SHIFT
Parameters C_v, R_1
Sensitivity curves ω versus R_1
C_v varies - $C = 10^{-9}$
$R = R_v = 1$ M
R_1 (ohms) x 10^5
10
9
8
7
6
5
4
3
2
1
1
2
3
4
5
6
7
8
9
10
A = 50
A = 40
A = 30
(rad/sec) x 10^3

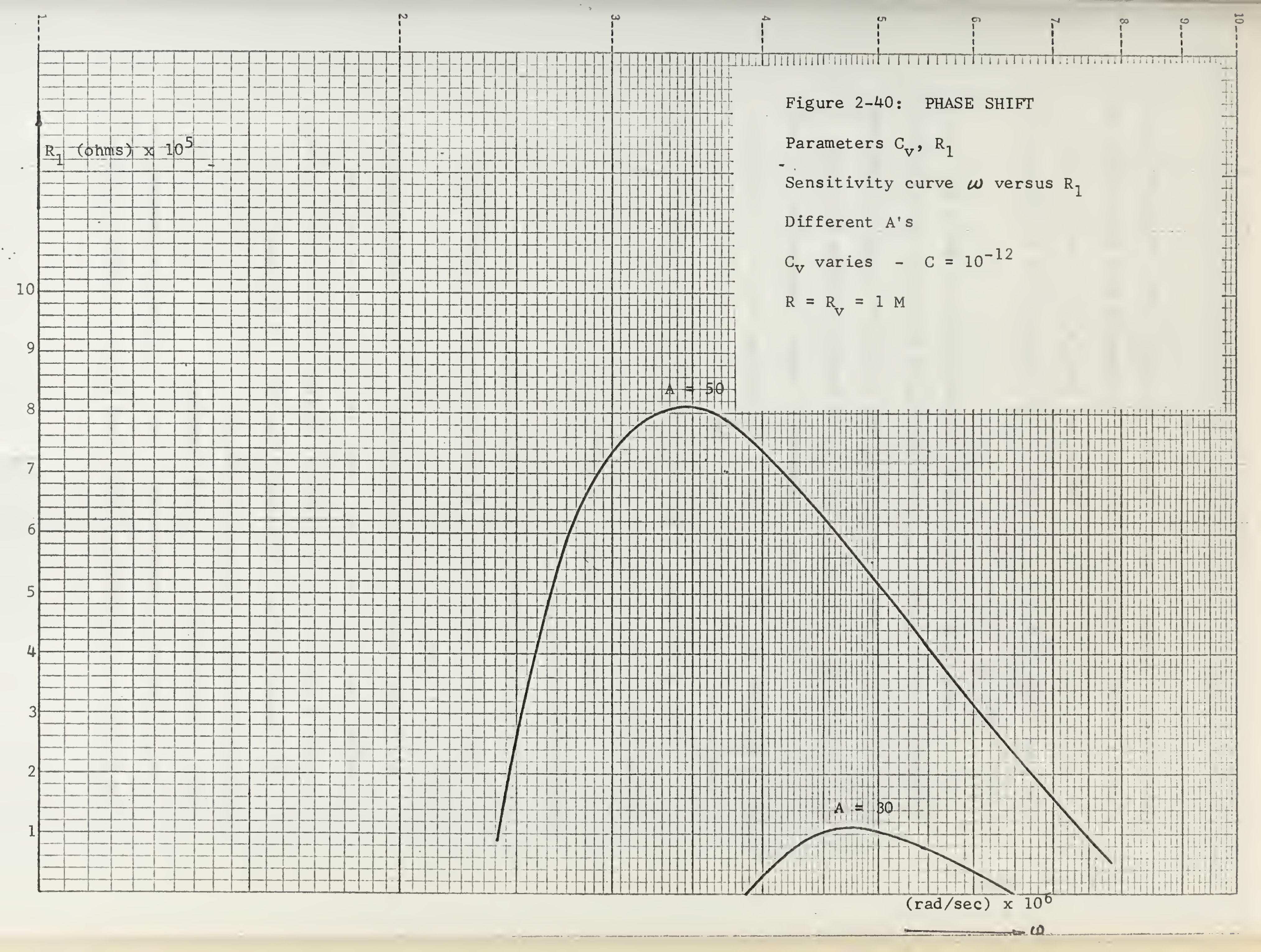


Figure 2-40: PHASE SHIFT
Parameters C_v, R_1
Sensitivity curve ω versus R_1
Different A's
C_v varies - $C = 10^{-12}$
$R = R_v = 1$ M
A = 50
A = 30
R_1 (ohms) x 10^5
(rad/sec) x 10^6
ω
1
2
3
4
5
6
7
8
9
10
1
2
3
4
5
6
7
8
9
10

the frequency curves, although they have the same shape, are raised up by $R_1 = 0.3 \times 10^5$ (Figures 2-39, 2-40). For example peak value at R_1 for $A = 30$ and $C = 10^{-9}$ fd is $0.8 \times 10^5 \Omega$ and for $C = 10^{-12}$ is 0.1×10^5 . Also peak value of R_1 for $A = 50$ and $C = 10^{-9}$ fd is $7.8 \times 10^5 \Omega$ and for $C = 10^{-12}$ fd, $R_1 = 8.1 \times 10^5 \Omega$.

There is a peak value of R_1 for every value of gain, i.e. $8.1 \times 10^5 \Omega$ for $A = 50$ (Figure 2.40). At this point frequency of oscillations is unstable because small change of R_1 may cause wide change of frequency which is wider for lower gains because the curve is flatter for lower gains and also this flat part is shifted to higher frequencies, i.e. for $A = 50$ the flat part is at about $\omega = .35$ krad/sec and for $A = 30$ it is shifted to $\omega = .48$ krad/sec (Figure 2-39).

For a given gain we can tune to different frequencies of oscillation with one value of R_1.

The higher the gain, the wider is the range of frequencies we can cover by varying R_1.

2-11. Sensitivity curves, ω versus C_v, with R_1 varying.

There is the same pattern of frequency variation for different values of C (Figures 2-41, 2-42).

For higher values of C_v (say $C_v \doteq 4 \times 10^{-10}$ fd) we have variation in frequency which is larger and larger as C_v gets smaller and smaller, i.e., from Figure 2-41 for a change of C_v from 20 to 16×10^{-10} fd for $A = 50$ we have a change in frequency from $\omega = .27$ krad/sec to .285 krad/sec while for a change of C_v from 12 to 8×10^{-10} fd we have a change in frequency from .31 to .355 krad/sec.

For values of C_v smaller than 4×10^{-10} fd change in frequency is more drastic as the curves get flatter and flatter at higher frequencies.

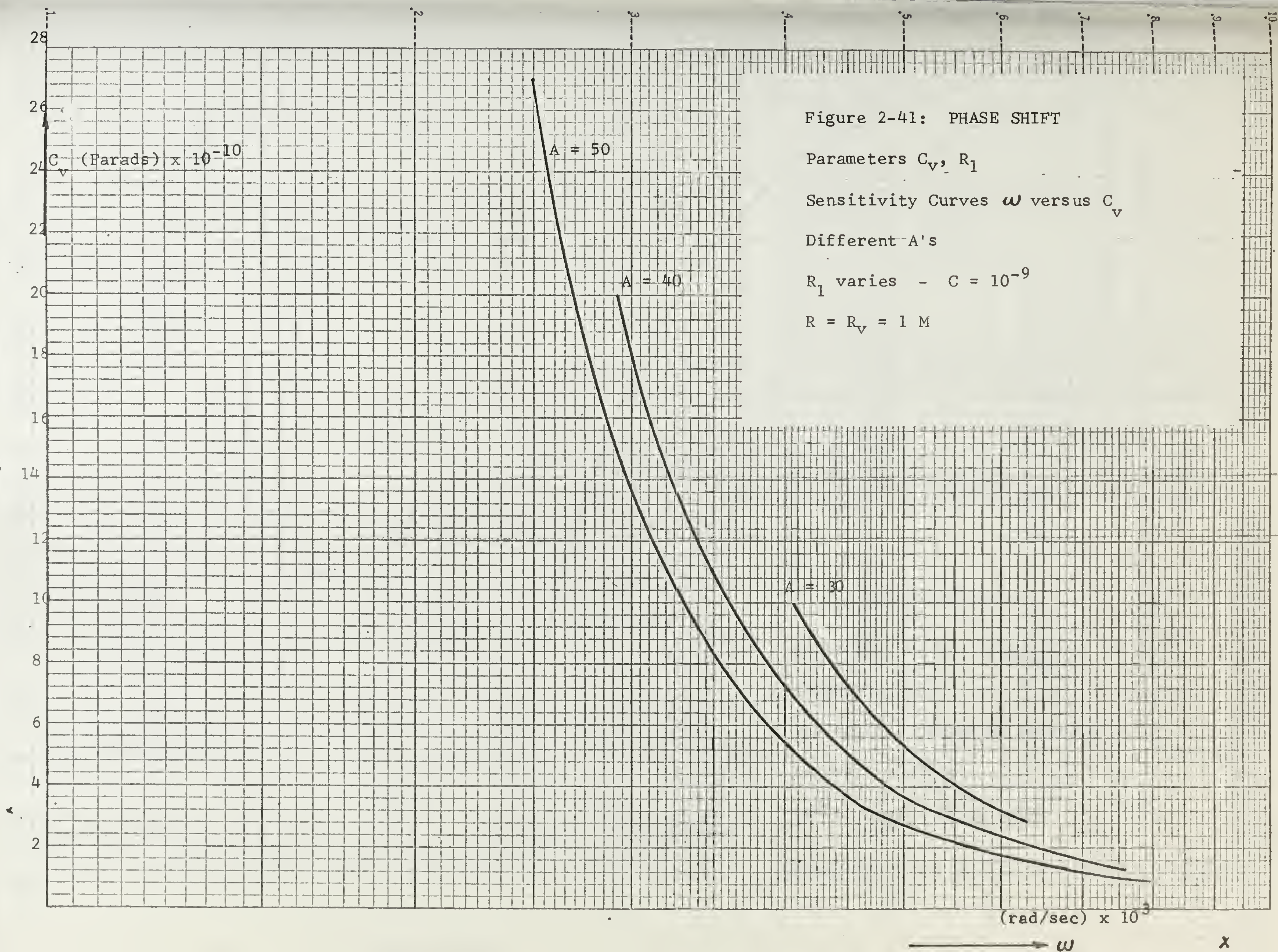


Figure 2-41: PHASE SHIFT
Parameters C_v, R_1
Sensitivity Curves ω versus C_v
Different A's
R_1 varies - $C = 10^{-9}$
$R = R_v = 1$ M
A = 50
A = 40
A = 30
C_v (Farads) x 10^{-10}
(rad/sec) x 10^3
ω
28
26
24
22
20
18
16
14
12
10
8
6
4
2
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0

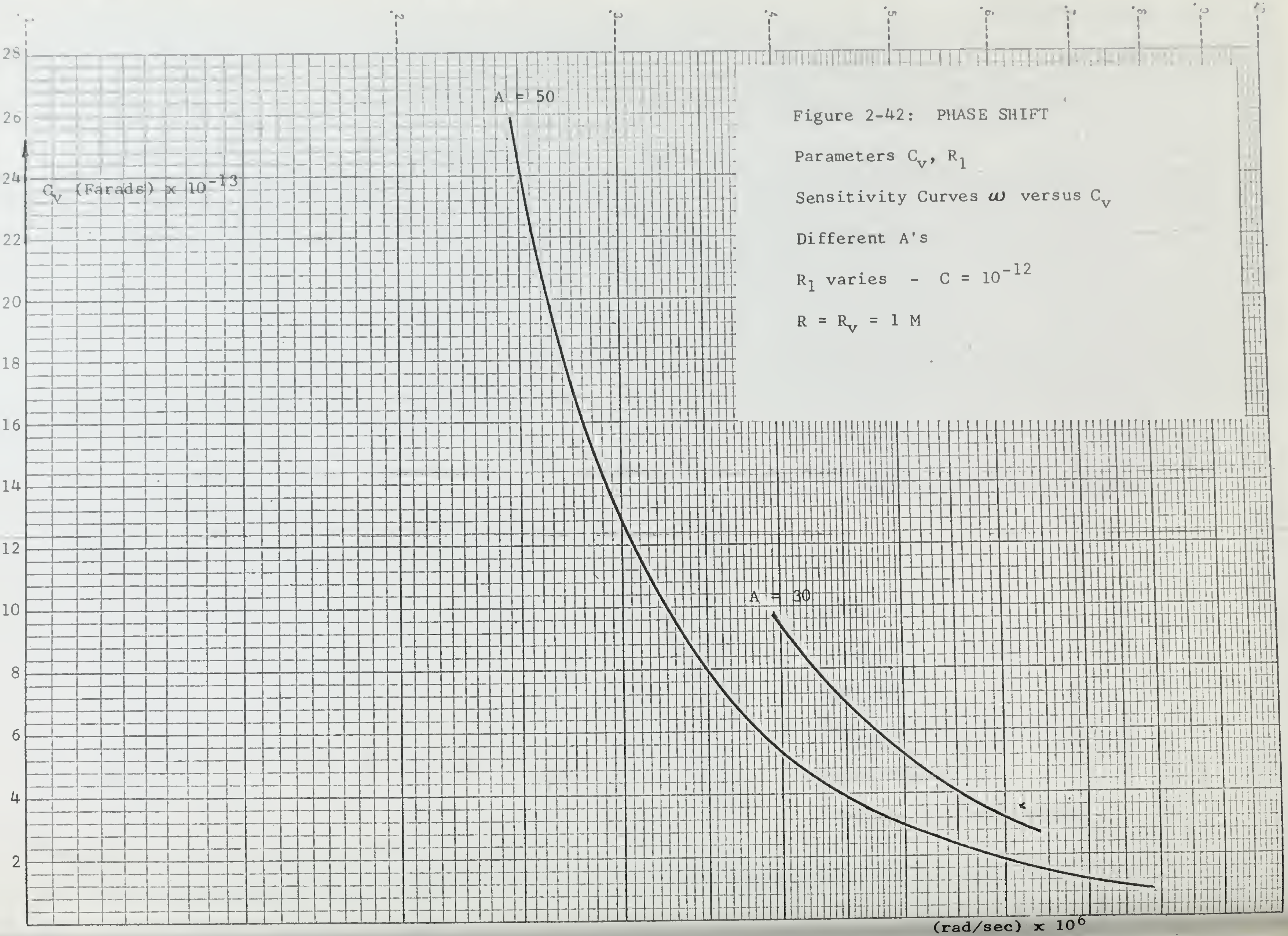
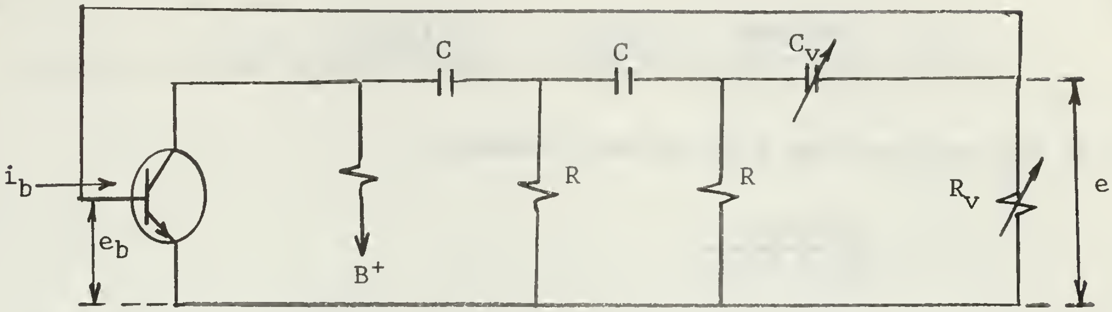


Figure 2-42: PHASE SHIFT
Parameters C_v, R_1
Sensitivity Curves ω versus C_v
Different A's
R_1 varies - $C = 10^{-12}$
$R = R_v = 1$ M
C_v (Farads) x 10^{-13}
A = 50
A = 30
(rad/sec) x 10^6
28
26
24
22
20
18
16
14
12
10
8
6
4
2
.2
.3
.4
.5
.6
.7
.8
.9

2-12. Transistor phase shift oscillator.

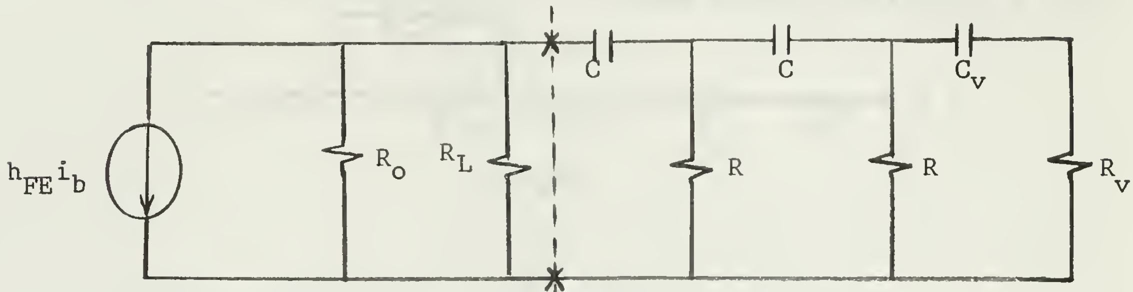
The same problem can be handled with transistors instead of tubes, with minor differences.



Transistor phase shift oscillator

Figure 2-43

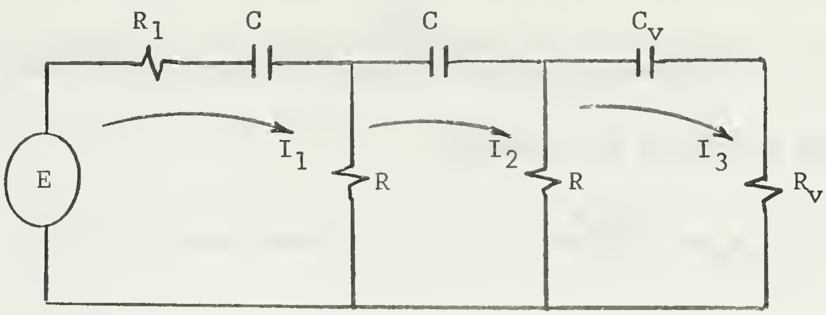
The equivalent circuit of Figure 2-43 is:



Transistor phase shift equivalent circuit

Figure 2-44

Using Thevenin equivalent to the left of points x-x we get:



Transistor phase shift equivalent circuit

Figure 2-45

where

$$R_1 = \frac{R_o R_L}{R_o + R_L}$$

R_o = output resistance

R_L = load resistance

$$E = -\frac{R_L R_o h_{FE} L_b}{R_L + R_o} \qquad \begin{aligned} i_b &= \text{base current} \\ h_{FE} &= \text{forward current gain} \end{aligned}$$

or

$$E = -\frac{R_L R_o h_{FE}}{R_L + R_o} \quad \frac{i_b r_n}{r_n} = -\frac{R_L R_o h_{FE}}{(R_L + R_o) r_n} e_b$$

So we can call voltage gain of the transistor

$$A = \frac{R_L R_o h_{FE}}{(R_L + R_o) r_n}$$

where r_n is the transistor base resistance plus the emitter resistance referred to base $r_n = r_b{}^1 + r_e(h_{FE} + 1)$ which can be seen from the transistor T equivalent circuit below.

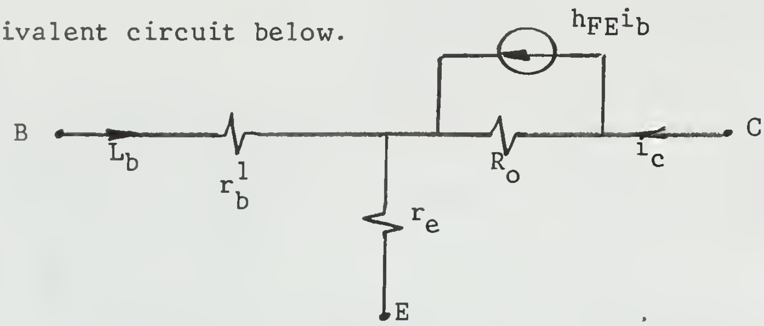


Figure 2-46. T-equivalent circuit

Solving the 3 loop equations of Figure 2-3 we get

$$I_3 = \frac{R^2 E}{-R^2(R+R_v+X_v)+(2R+X)(R_1+R+X)(R+R_v+X_v)+R^2(R_1+R+X)}$$

Voltage at the output of the circuit

$$e = I_3 R_v = \frac{R^2 R_v E}{D} \qquad D \equiv \text{denominator of (7)}$$

or

$$e = \frac{R^2 R_v}{D}(-Ae_b) \qquad \frac{e}{e_b} = -\frac{R^2 R_v}{D} A$$

but

$$\frac{e}{e_b} = AB \qquad \text{so} \quad AB = -\frac{R^2 R_v}{D} A$$

feedback factor $B = -\frac{R^2 R_V}{D}$

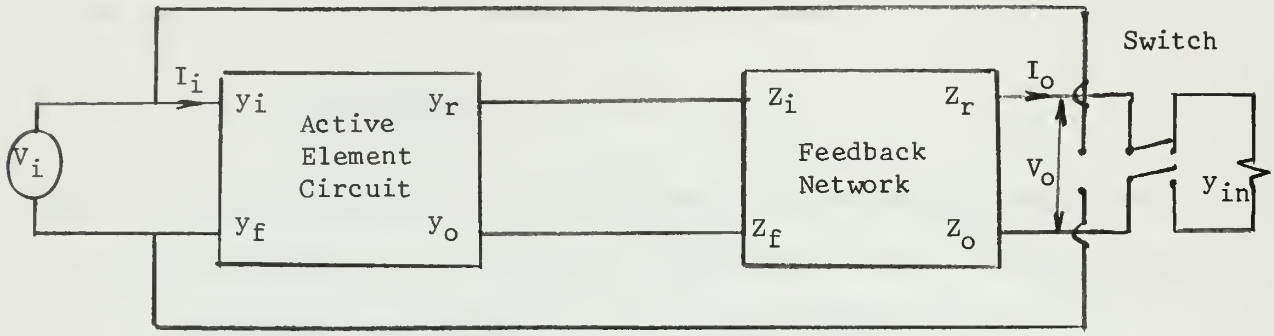
and following the same technique we come out with the same characteristic equation as in the tube oscillator.

The same results so far hold for transistor oscillators also.

3. General Approach in Deriving Characteristic Equations

We are going to further study the most common existing feed back oscillator from the parameter plane point of view.

The generalized representation of a feed back oscillator [2] is as follows:



Feedback oscillator

Figure 3-1

which is a typical 2 port network where we use y parameters for the active element's circuit because we easier associate y parameters with the parameters of the circuit, say gm, $1/r_p$ etc. For the feedback network, conversely, the Z parameters are used, because they are more readily available.

In the circuit above, the feedback loop is opened and the active element circuit, which is an amplifier, is fed by an external voltage V_i. Then the input admittance of the amplifier, y_{in}, which normally loads the output of the feedback network is placed after feedback. This situation now represents the case of an amplifier with feedback. But if the amplifier and feedback components are so adjusted that $V_o = V_i$ and $I_o = I_i$ in magnitude and phase, then we can put the switch to the left side and remove source V_i. So we have the equivalent circuit.

Feedback oscillator equivalent circuit

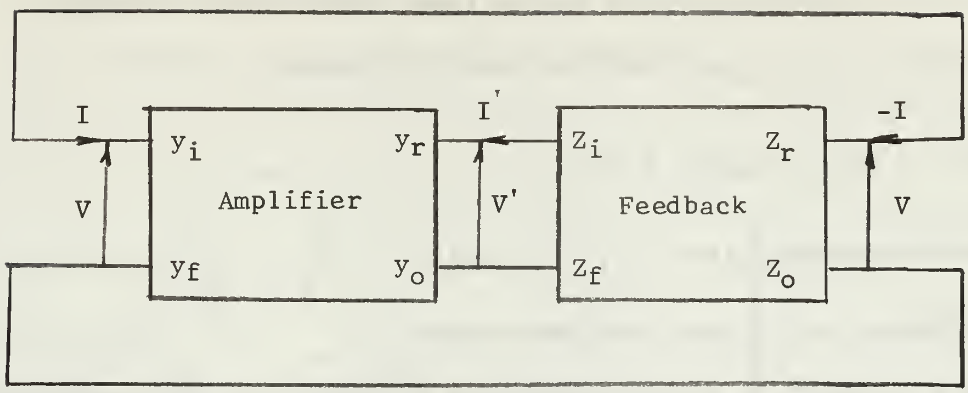


Figure 3-2

The fact that voltage at the output port is equal to the input voltage before the feedback loop is closed, is equivalent to the fact that the feedback in a closed system must be regenerative and closed loop amplification under steady state conditions must be equal to unity which gives the conditions for oscillations.

The conditions that output voltage and current are equal to input voltage and current in magnitude and phase, are described by the following equations, for a linearized amplifier circuit.

$$I = Vy_i + V'y_r$$

$$I' = Vy_f + V'y_o$$

$$V = -I'Z_i - IZ_r$$

$$V' = -I'Z_f - IZ_o$$

Simultaneous solution of these 4 equations lead to the relation:

$$y_fZ_f + y_iZ_o + y_oZ_i + y_rZ_r + (y_iy_o - y_fy_r)(Z_iZ_o - Z_fZ_r) + 1 = 0$$

Since $Z_r = Z_f$ for a passive network and $y_f \gg y_r$ for an amplifier, y_rZ_r is negligible so the equation becomes

$$y_fZ_f + y_iZ_o + y_oZ_i + (y_iy_o - y_fy_r)(Z_iZ_o - Z_fZ_r) + 1 = 0$$

In the case of common cathode vacuum tubes where there is no grid current flowing, y_i and y_r are zero so equation becomes

$$(18) \qquad y_f Z_f + y_o Z_i + 1 = 0$$

In a common-cathode circuit $y_f = g_m$ and $y_o = \frac{1}{r_p} + \frac{1}{R_L}$ where r_p is plate resistance and R_L the load resistance.

The above relation is nothing more than the condition for oscillation AB=1 which by itself is the characteristic equation of the whole circuit.

We are going to use this relation in deriving the characteristic equation in the S plane of representative types of oscillators.

4. Colpitt's Oscillator

4-1. Derivation of characteristic equation.

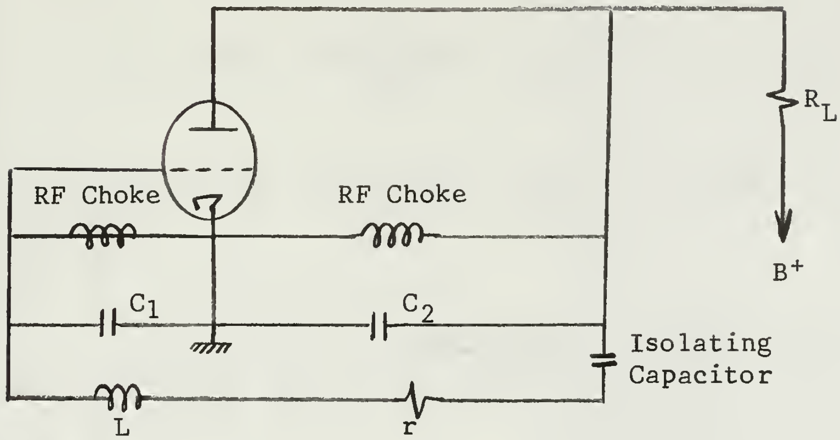


Figure 4-1. Colpitt's oscillator

Characteristic equation:

$$y_f Z_f + y_o Z_i + 1 = 0$$

$$y_f = g_m$$

$$Z_f = -\frac{1}{\omega^2 C_1 C_2 Z} = \frac{1}{S^2 C_1 C_2 Z}$$

$$Z_i = -\frac{1}{Z}\left[\frac{1}{\omega^2 C_1 C_2} + j\left(\frac{r}{\omega C_2}\right) - \frac{L}{C_2}\right]$$

$$= \frac{1}{Z}\left[\frac{1}{S^2 C_1 C_2} + \frac{r}{SC_2} + \frac{L}{C_2}\right] = \frac{1}{Z}\ \frac{1 + SC_1 r + S^2 C_1 L}{S^2 C_1 C_2}$$

$$Z = r + SL + \frac{1}{SC_1} + \frac{1}{SC_2} + \frac{S^2 r C_1 C_2 + S^3 C_1 C_2 L + SC_2 + SC_1}{S^2 C_1 C_2}$$

$$= \frac{S^2 C_1 C_2 L + S\, r C_1 C_2 + C_1 + C_2}{SC_1 C_2}$$

Plug into the characteristic equation:

$$g_m \frac{1}{S^2 C_1 C_2 Z} + y_o\left[\frac{1}{Z}\ \frac{(1 + SC_1 r + S^2 C_1 L)}{S^2 C_1 C_2}\right] + 1 = 0$$

$$g_m + y_o(S^2C_1L + SC_1r + 1) + S^2C_1C_2Z = 0$$

$$g_m + S^2y_oC_1L + Sy_oC_1r + y_o + S^2C_1C_2\left[\frac{S^2C_1C_2L + S\ rC_1C_2 + C_1 + C_2}{SC_1C_2}\right] = 0$$

$$g_m + S^2y_oC_1L + Sy_oC_1r + y_o + S^3C_1C_2L + S^2rC_1C_2 + SC_1 + SC_2 = 0$$

$$S^3C_1C_2L + S^2(y\ C_1L + rC_1C_2) + S\ (y_oC_1r + C_1 + C_2) + (g_m + y_o) = 0$$

where

$$y_o = \frac{1}{Z_o} = \frac{1}{r_p//R_L} = \frac{r_p + R_L}{r_pR_L}$$

divide by y_o and set gain $A = \frac{y_m}{y_o}$

$$S^3\frac{C_1C_2L}{y_o} + S^2(C_1L + \frac{rC_1C_2}{y_o}) + S(C_1r + \frac{C_1 + C_2}{y_o}) + (A + 1) = 0$$

4-2. Parameter plane curves C_1 versus L for different gains.

Giving different values to the tube circuit parameters:

<u>For A = 30</u>

$y_o = 10^{-4}$ mhos

$r = 100\ \Omega$

$C_2 = 10^{-12}$ fd

The characteristic equation becomes:

$$S^3(10^{-8}C_1L) + S^2(C_1L + 10^{-6}C_1) + S(1.01 \times 10^4C_1 + 10^{-8}) + 31 = 0$$

<u>For A = 40</u>

$$S^3(10^{-8}C_1L) + S^2(C_1L + 10^{-6}C_1) + S(1.01 \times 10^4C_1 + 10^{-8}) + 41 = 0$$

<u>For a = 50</u>

$$S^3(10^{-8}C_1L) + S^2(C_1L + 10^{-6}C_1) + S(1.01 \times 10^4C_1 + 10^{-8}) + 51 = 0$$

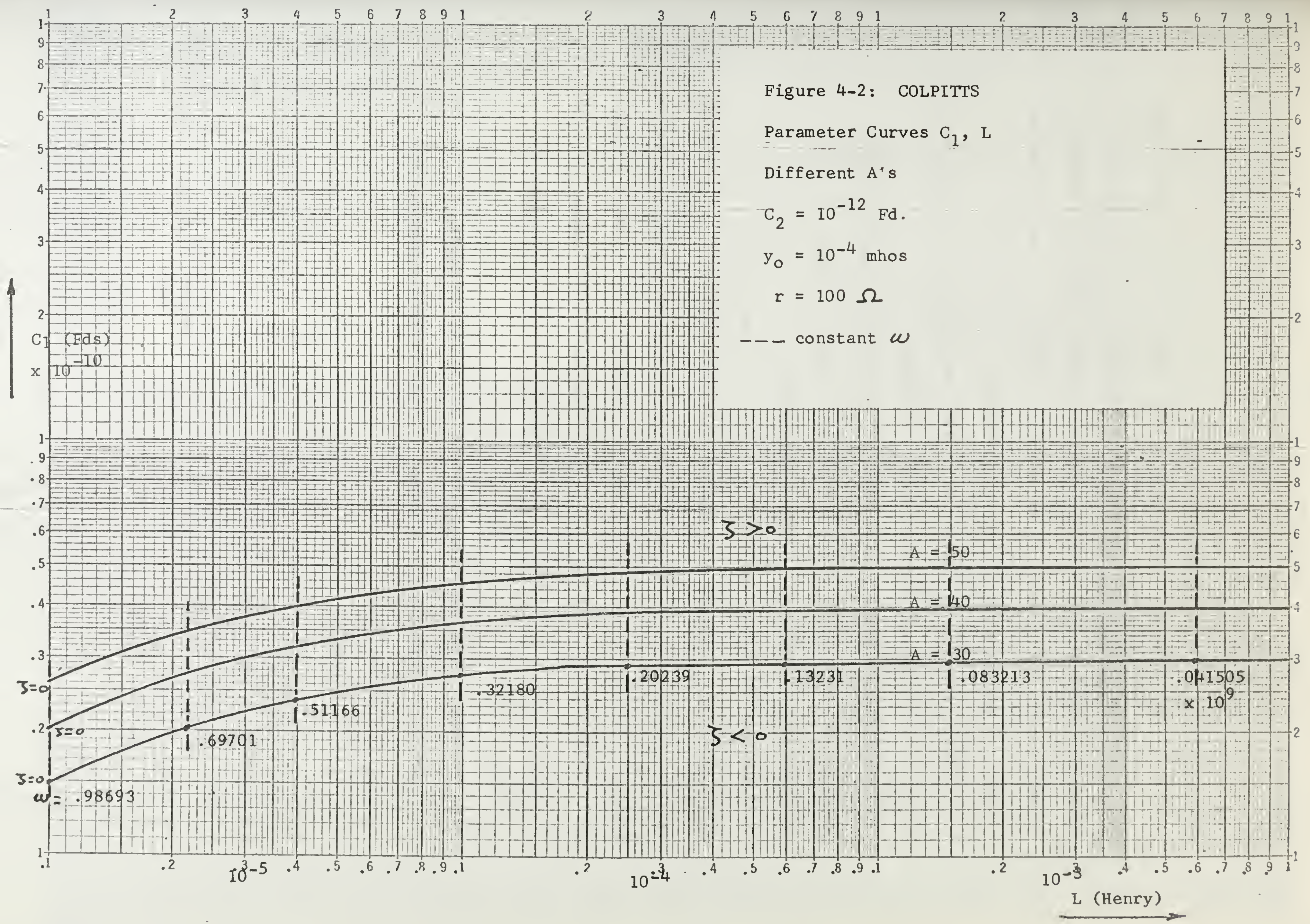


Figure 4-2: COLPITTS

Parameter Curves C_1, L

Different A's

$C_2 = 10^{-12}$ Fd.

$y_o = 10^{-4}$ mhos

$r = 100\ \Omega$

--- constant ω

Results for Colpitt's; Parameters C_1, L

If we fix capacitor C_2 of the feedback circuit at the value of 1$\mu\mu$f and have as varying parameters the other capacitor C_1 and L, for low frequencies such as $\omega < 30 \times 10^6$ rad/sec, we have control of frequency by only varying inductance L. For example, for gain A = 40, C_1 must be fixed to 39 $\mu\mu$f in order to have oscillations.

This value of C_1 is higher, the higher the gain is, i.e., for gain A = 30, C_1 = 29 $\mu\mu$f and for A = 50, C_1 49 $\mu\mu$f.

For higher frequencies ($\omega > 30 \times 10^6$ rad/sec) C_1 remains no more constant, but it must be reduced while L is reduced in order to have oscillations at higher and higher frequencies.

Constant ω curves are vertical. So if the gain changes and L is constant, then by varying C we stay at the same frequency. So by virtue of this fact, we can detect any change in gain by the change of C_1 tuned for a certain frequency. For example, for gain A = 50, we set L = .6mh and C_1 = 49 $\mu\mu$f in order to have oscillations at 41.505 Mrad/sec, but if C_1 tunes at C = 39 $\mu\mu$f for the same frequency, it means that gain is A = 40 and not 50.

4-3. Parameter plane curves C_2 versus L for different gains.

For the following typical values of the circuit parameters:

$y_o = 10^{-4}$ mhos

$r = 100\ \Omega$

$C_1 = 10^{-12}$ fd

Characteristic equation:

$$S^3 \frac{C_1C_2L}{y_o} + S^2\left(C_1L + \frac{rC_1C_2}{y_o}\right) + S\left(C_1r + \frac{C_1 + C_2}{y_o}\right) + (A + 1) = 0$$

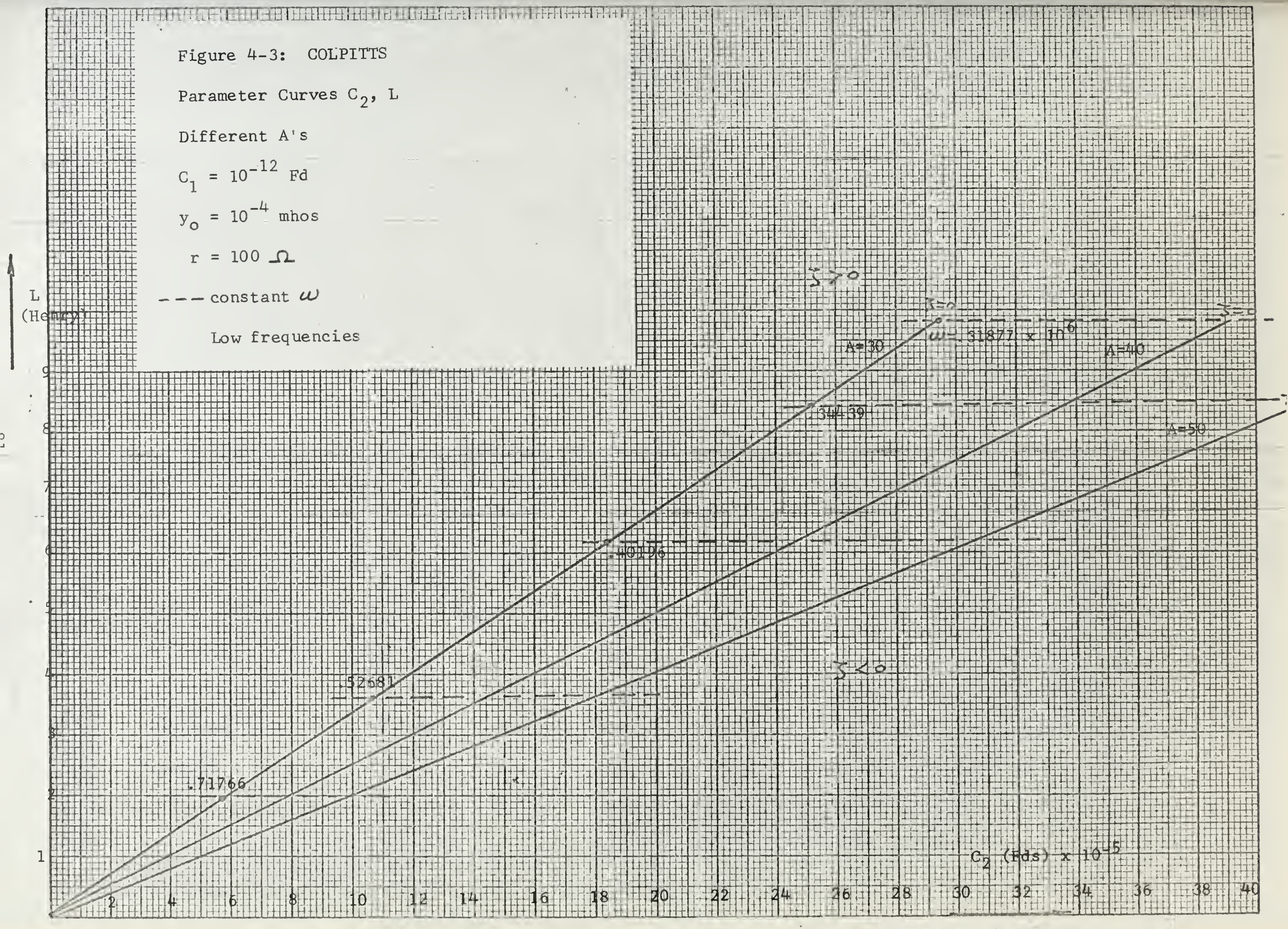


Figure 4-3: COLPITTS

Parameter Curves C_2, L

Different A's

$C_1 = 10^{-12}$ Fd

$y_o = 10^{-4}$ mhos

r = 100 Ω

- - - constant ω

Low frequencies

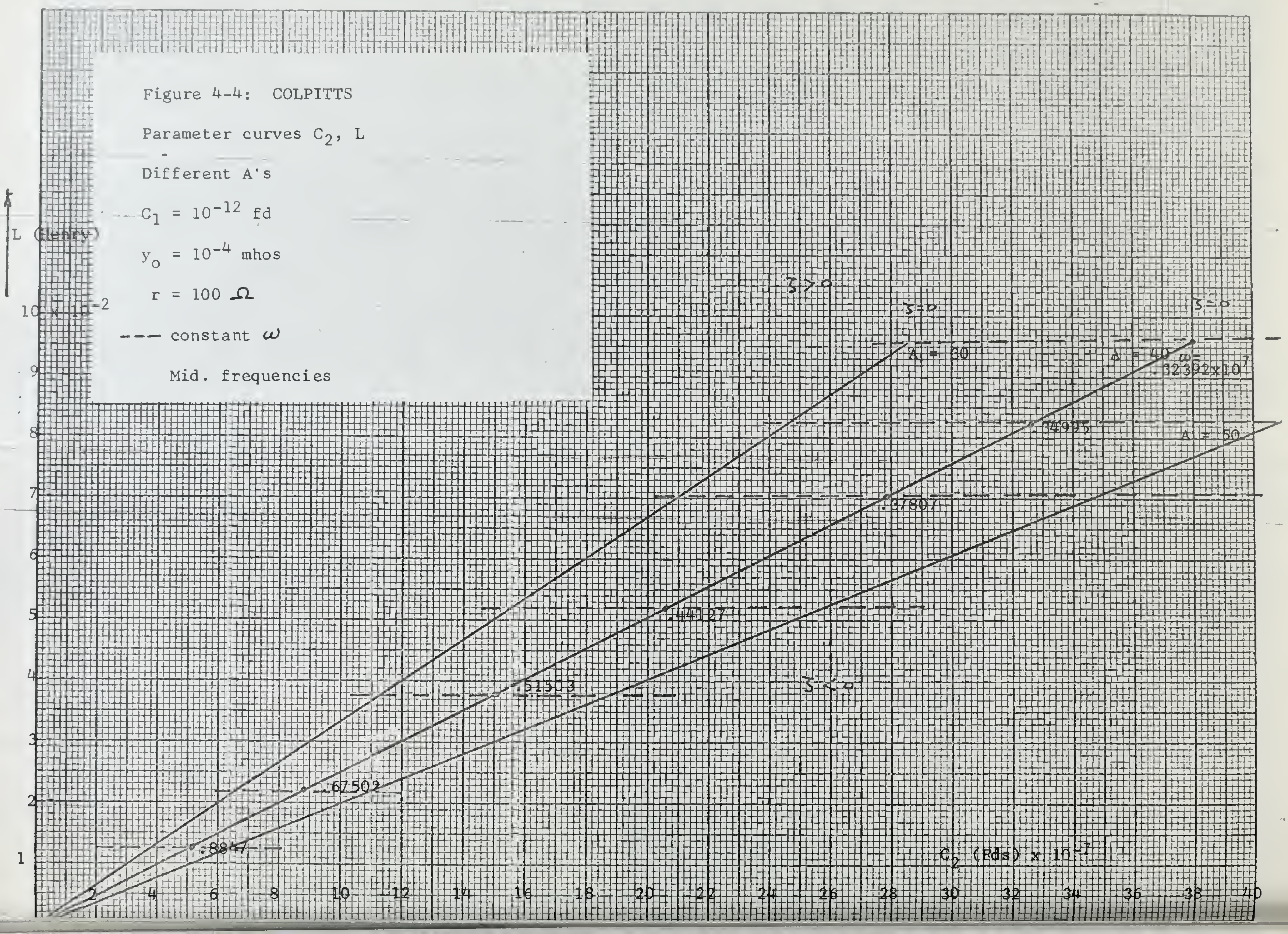


Figure 4-4: COLPITTS
Parameter curves C_2, L
Different A's
$C_1 = 10^{-12}$ fd
$y_o = 10^{-4}$ mhos
r = 100 Ω
--- constant ω
Mid. frequencies
L (Henry)
10×10^{-2}
9
8
7
6
5
4
3
2
1
C_2 (Fds) x 10^{-7}
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
A = 30
A = 40
A = 50
$\zeta > 0$
$\zeta = 0$
$\zeta < 0$
$\omega = .32392 \times 10^7$
.34995
.37807
.44127
.51503
.67502
.8847

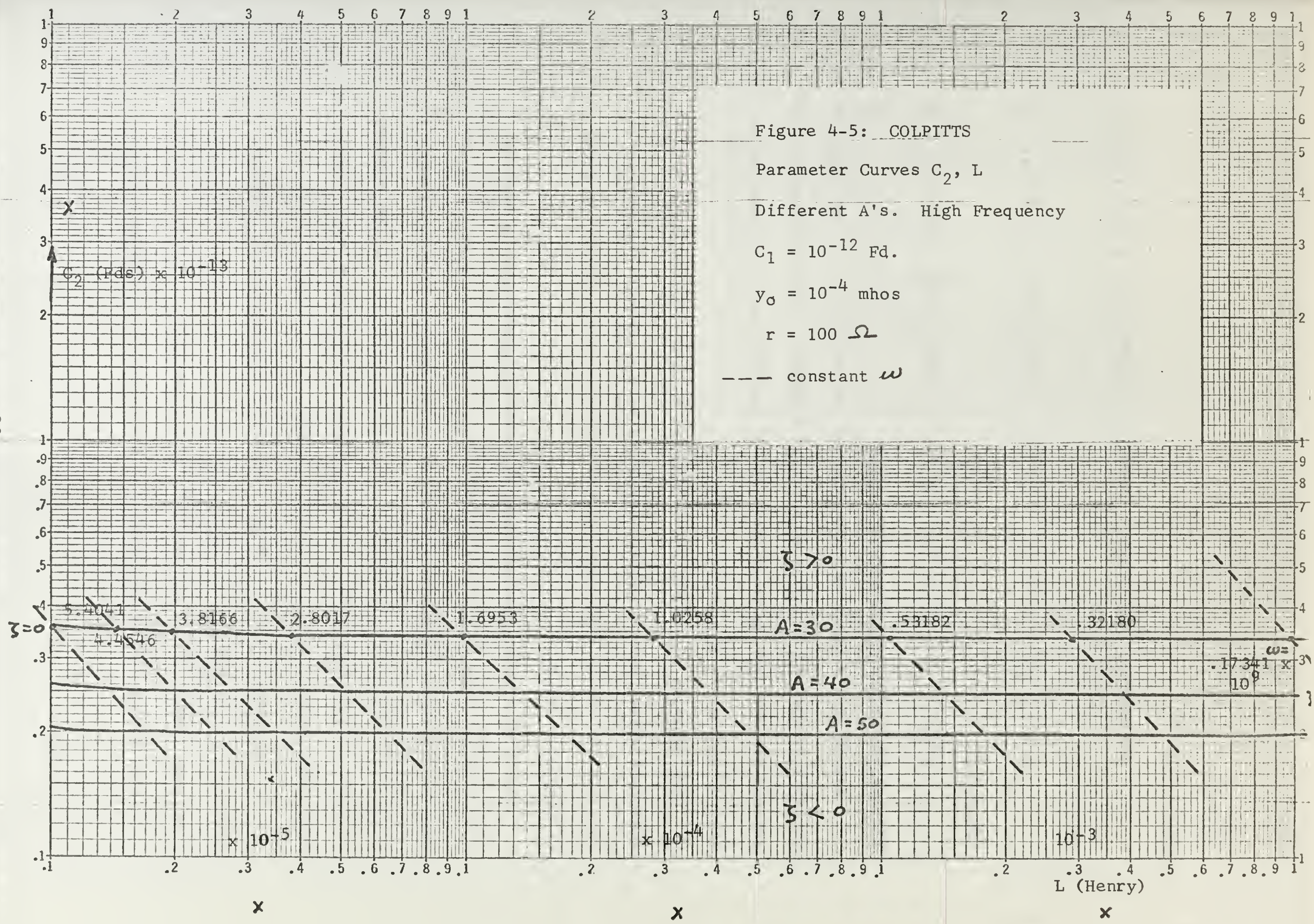


Figure 4-5: COLPITTS

Parameter Curves C_2, L

Different A's. High Frequency

$C_1 = 10^{-12}$ Fd.

$y_o = 10^{-4}$ mhos

$r = 100\ \Omega$

--- constant ω

becomes:

For A = 30

$$S^3(10^{-8}C_2L) + S^2(10^{-6}C_2 + 10^{-12}L) + S(10^4C_2+1.01 \times 10^{-8}) + 31 = 0$$

For A = 40

$$S^3(10^{-8}C_2L) + S^2(10^{-6}C_2 + 10^{-12}L) + S(10^4C_2+1.01 \times 10^{-8}) + 41 = 0$$

For A = 50

$$S^3(10^{-8}C_2L) + S^2(10^{-6}C_2 + 10^{-12}L) + S(10^4C_2+1.01 \times 10^{-8}) + 51 = 0$$

Results for Colpitt's: Parameters C_2, L

Now if we fix the other capacitor of the circuit C_1 = 1 μμf and have variable parameters C_2 and L we see from Figures 4-3 and 4-4 that for low and mid frequencies (up to $\omega = 0.1 \times 10^8$ rad/sec) we have linear variation of parameters C_2, L.

The constant ω curves are horizontal so again for constant L and different gains we must adjust only C_2 to have a certain frequency of oscillations and by this means we can detect any gain changes in terms of frequency change.

For high frequencies parameter C_2 remains constant while varying only L we can tune to any frequency (Figure 4-5). This result can be used to detect any variation of L in terms of frequency if C_2 is supposed to be constant. For example for A = 50, if C_2 = 0.02 μμf suppose we drift from frequency ω = 321.8 Mrad/sec to 531.8 Mrad/sec it means that L has changed from 0.49 mh to 0.18 mh.

C_2 has the value of 0.035 μμf for A = 30 and reduces for higher gains, C_2 = 0.02 μμf for A = 50.

4-4. Sensitivity curves ω versus C_1 with L varying.

From Figure 4-6 we conclude that for changes of C_1 we have almost

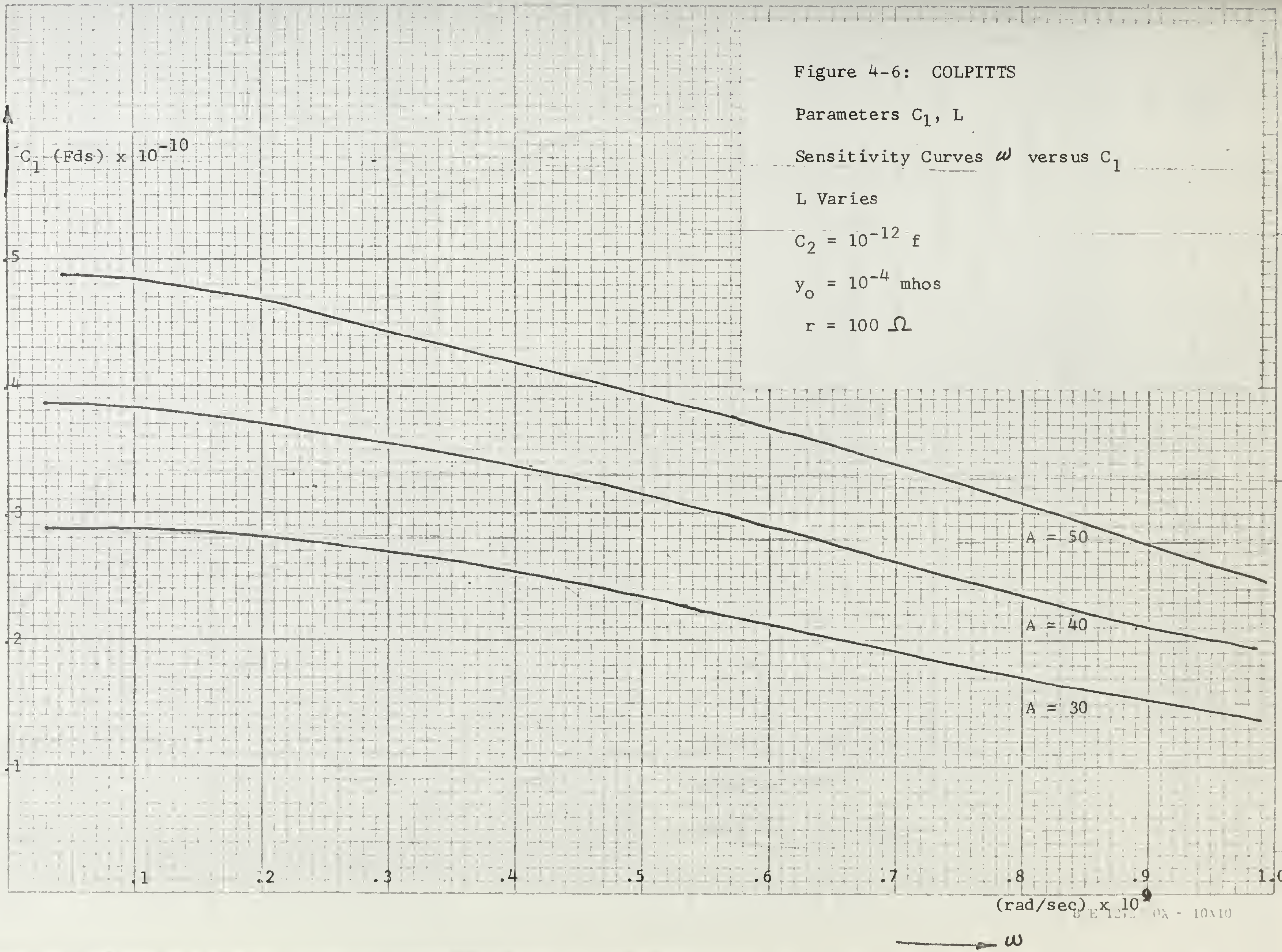


Figure 4-6: COLPITTS

Parameters C_1, L

Sensitivity Curves ω versus C_1

L Varies

$C_2 = 10^{-12}$ f

$y_o = 10^{-4}$ mhos

$r = 100\ \Omega$

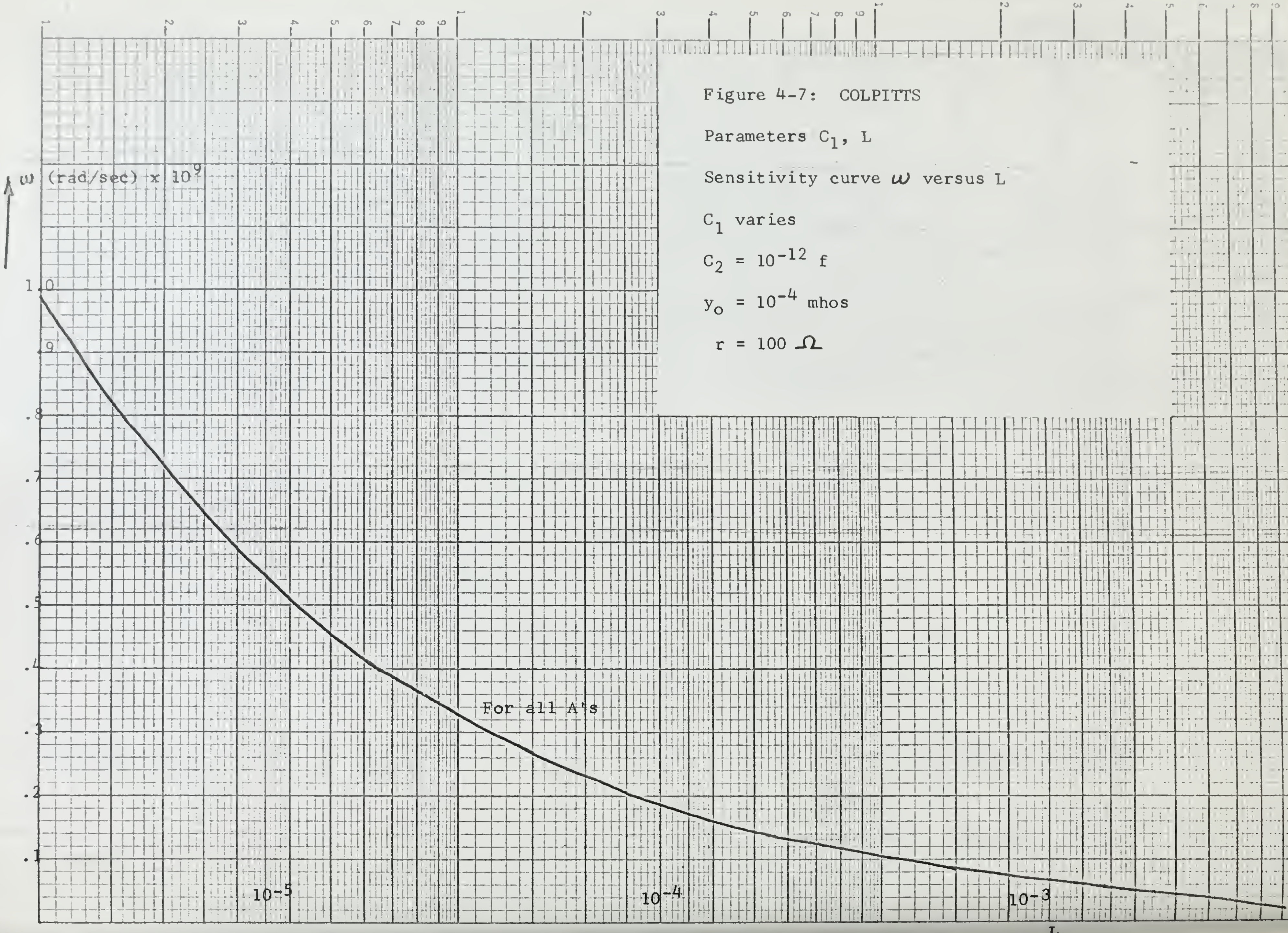


Figure 4-7: COLPITTS
Parameters C_1, L
Sensitivity curve ω versus L
C_1 varies
$C_2 = 10^{-12}$ f
$y_o = 10^{-4}$ mhos
$r = 100\ \Omega$
ω (rad/sec) x 10^9
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
For all A's
10^{-5}
10^{-4}
10^{-3}
L

linear change in frequency. The curves present the same shape for different gains.

4-5. Sensitivity curves ω versus L with C_1 varying.

For all the gains we have the same curve so frequency versus L curve is independent of gain (Figure 4-7).

As L gets higher and higher values, of the order of 10^{-3} h then the curve is getting flatter and flatter so a wide change in L causes only small change in frequency.

As L is getting lower and lower values, of the order of 10^{-5} h, then the lower the value of L the wider the change in frequency, i.e. change of L from $.5 \times 10^{-5}$ h to $.4 \times 10^{-5}$ h causes a change in frequency from $.45 \times 10^9$ to $.5 \times 10^9$ rad/sec., while a change of L from $.3 \times 10^{-5}$ to $.2 \times 10^{-5}$ h causes a change in frequency from $\omega = .59 \times 10^9$ to $\omega = .72 \times 10^9$ rad/sec.

4-6. Sensitivity curves ω versus C_2 with L varying.

For low and middle frequencies, or for ω up to 1×10^7 rad/sec, the curves for different gains give the same pattern of frequency variation with C_2 as shown in Figures 4-8 and 4-9.

As we change C_2 from a high value to lower and lower values we have a higher and higher change in frequency, i.e., from Figure 4-8 for A = 40 for a change of C_2 from 25×10^{-5} to 20×10^{-5} fd there is a change is frequency from .4 krad/sec to .445 Mrad/sec while a change of C_2 from 15×10^{-5} to 10×10^{-5} fd gives a change in frequency from .52 Mrad/sec to .63 Mrad/sec. The change in frequency is higher as we approach the flattening part of the curves.

For higher frequencies like $\omega = 10^9$ rad/sec with a constant C_2 we can produce a whole range of frequencies (by varying only the other para-

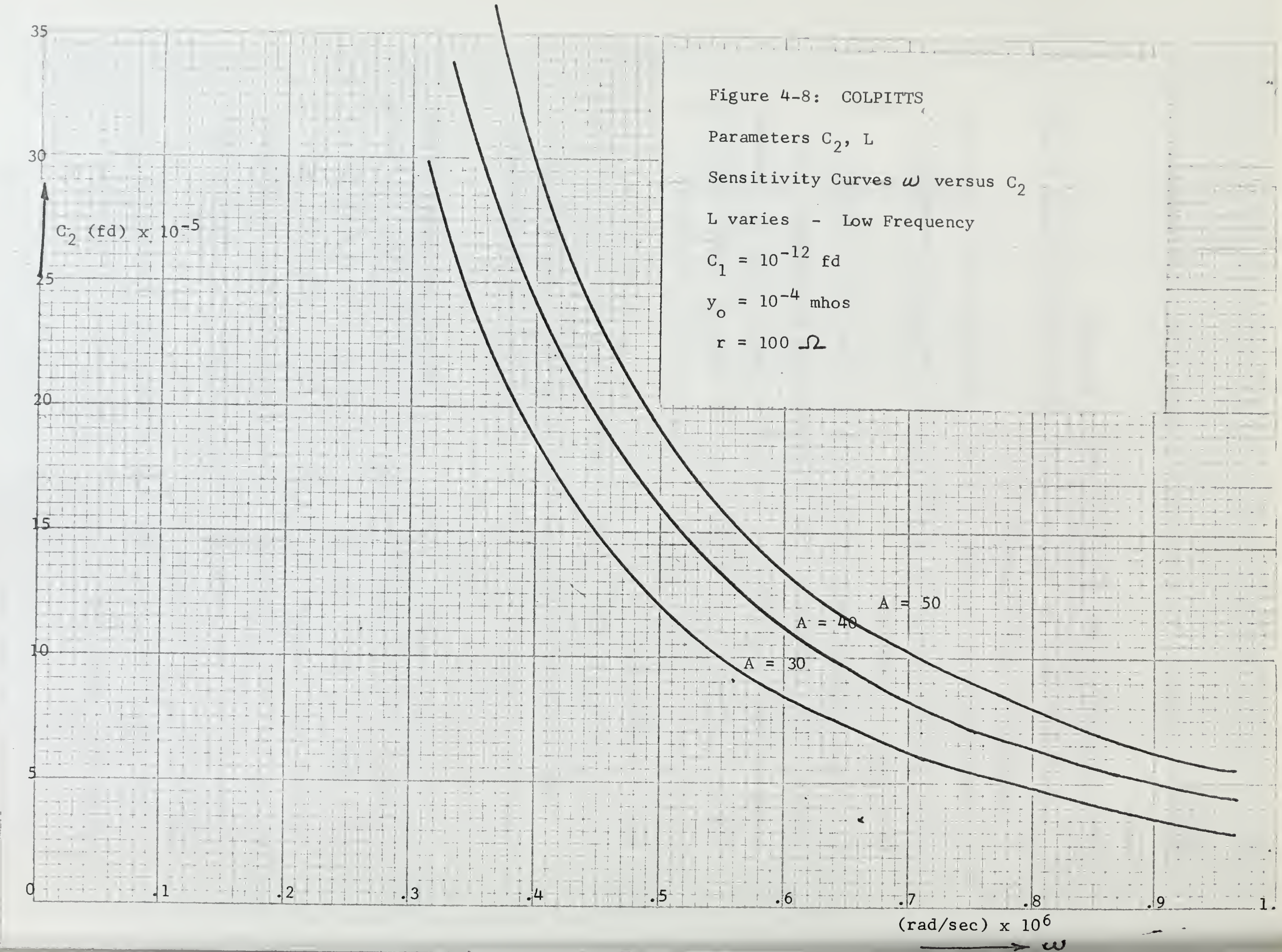


Figure 4-8: COLPITTS

Parameters C_2, L

Sensitivity Curves ω versus C_2

L varies - Low Frequency

$C_1 = 10^{-12}$ fd

$y_o = 10^{-4}$ mhos

$r = 100\ \Omega$

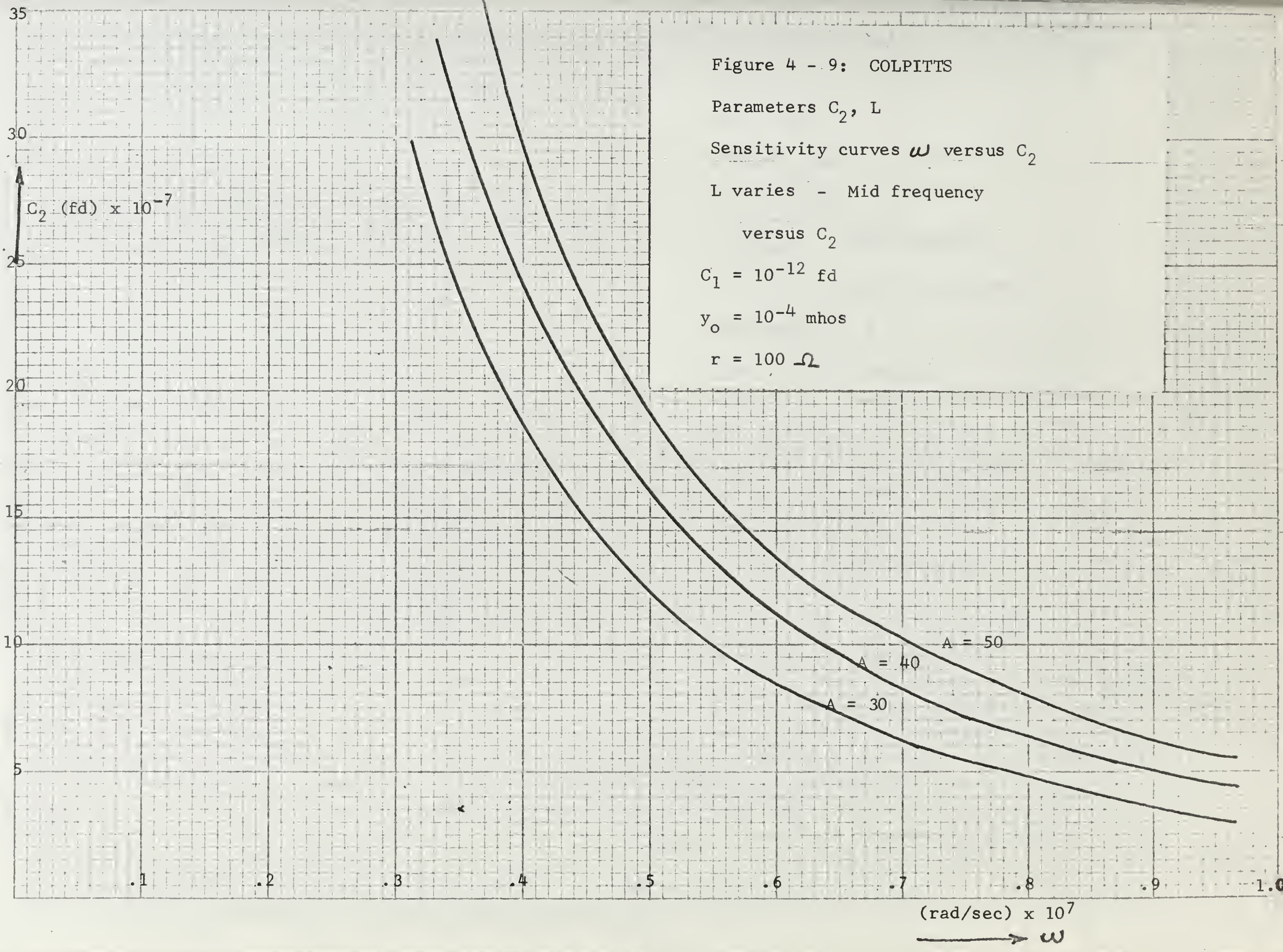


Figure 4 - 9: COLPITTS

Parameters C_2, L

Sensitivity curves ω versus C_2

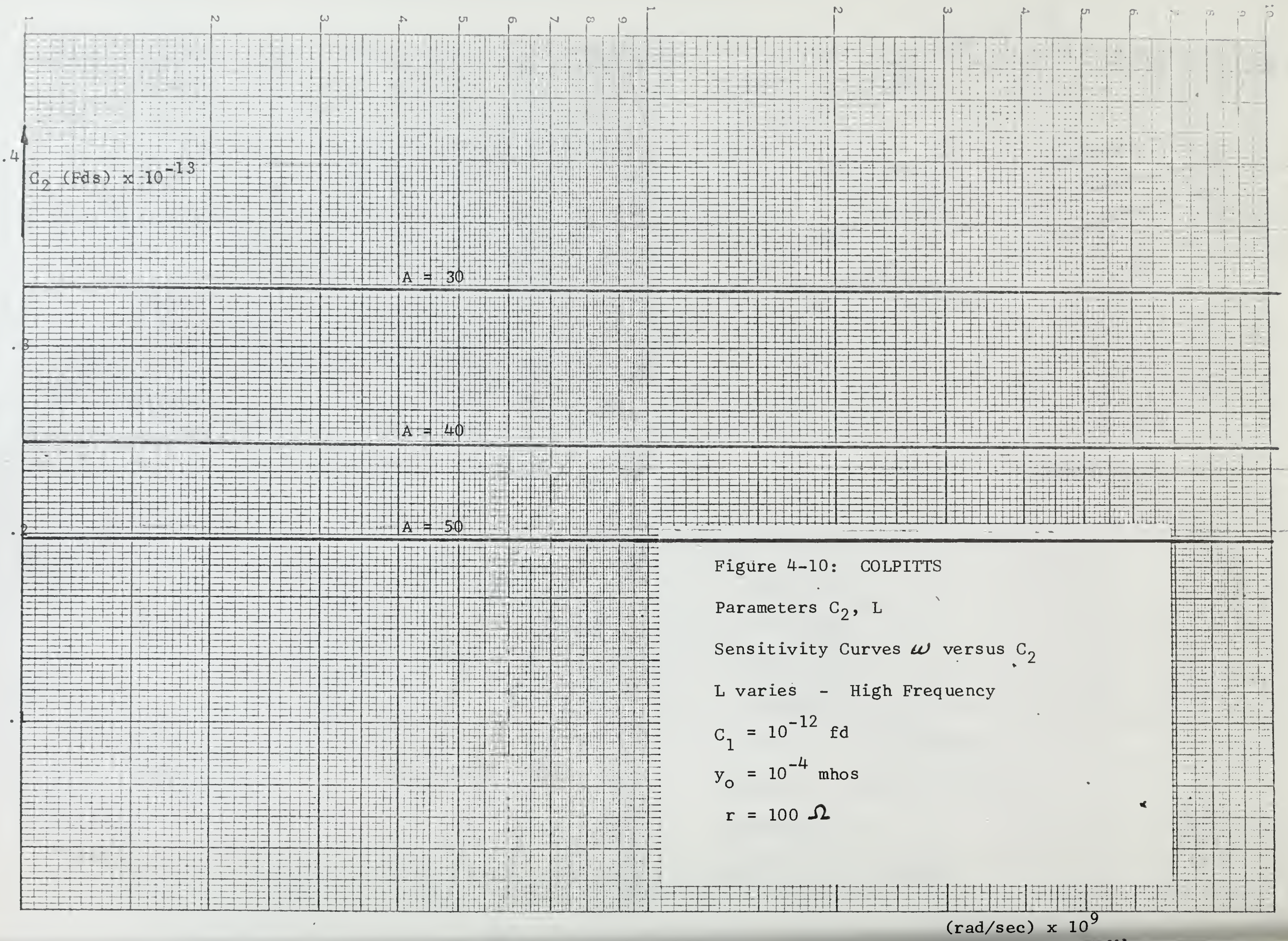
L varies - Mid frequency

versus C_2

$C_1 = 10^{-12}$ fd

$y_o = 10^{-4}$ mhos

$r = 100\ \Omega$



C_2 (Fds) x 10^{-13}
.4
.3
.2
.1
A = 30
A = 40
A = 50
Figure 4-10: COLPITTS
Parameters C_2, L
Sensitivity Curves ω versus C_2
L varies - High Frequency
$C_1 = 10^{-12}$ fd
$y_o = 10^{-4}$ mhos
$r = 100\ \Omega$
(rad/sec) x 10^9

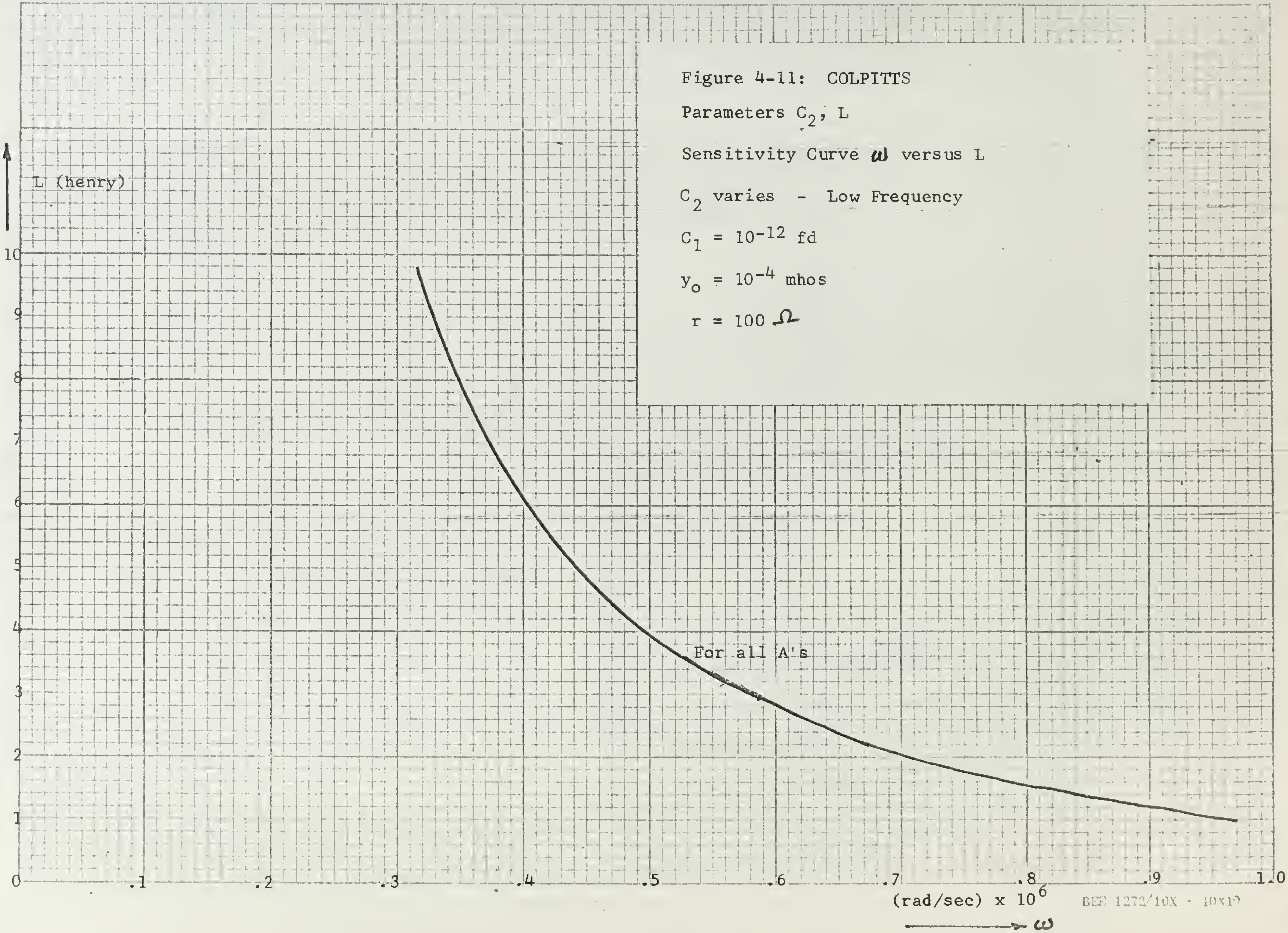


Figure 4-11: COLPITTS

Parameters C_2, L

Sensitivity Curve ω versus L

C_2 varies - Low Frequency

$C_1 = 10^{-12}$ fd

$y_o = 10^{-4}$ mhos

$r = 100\ \Omega$

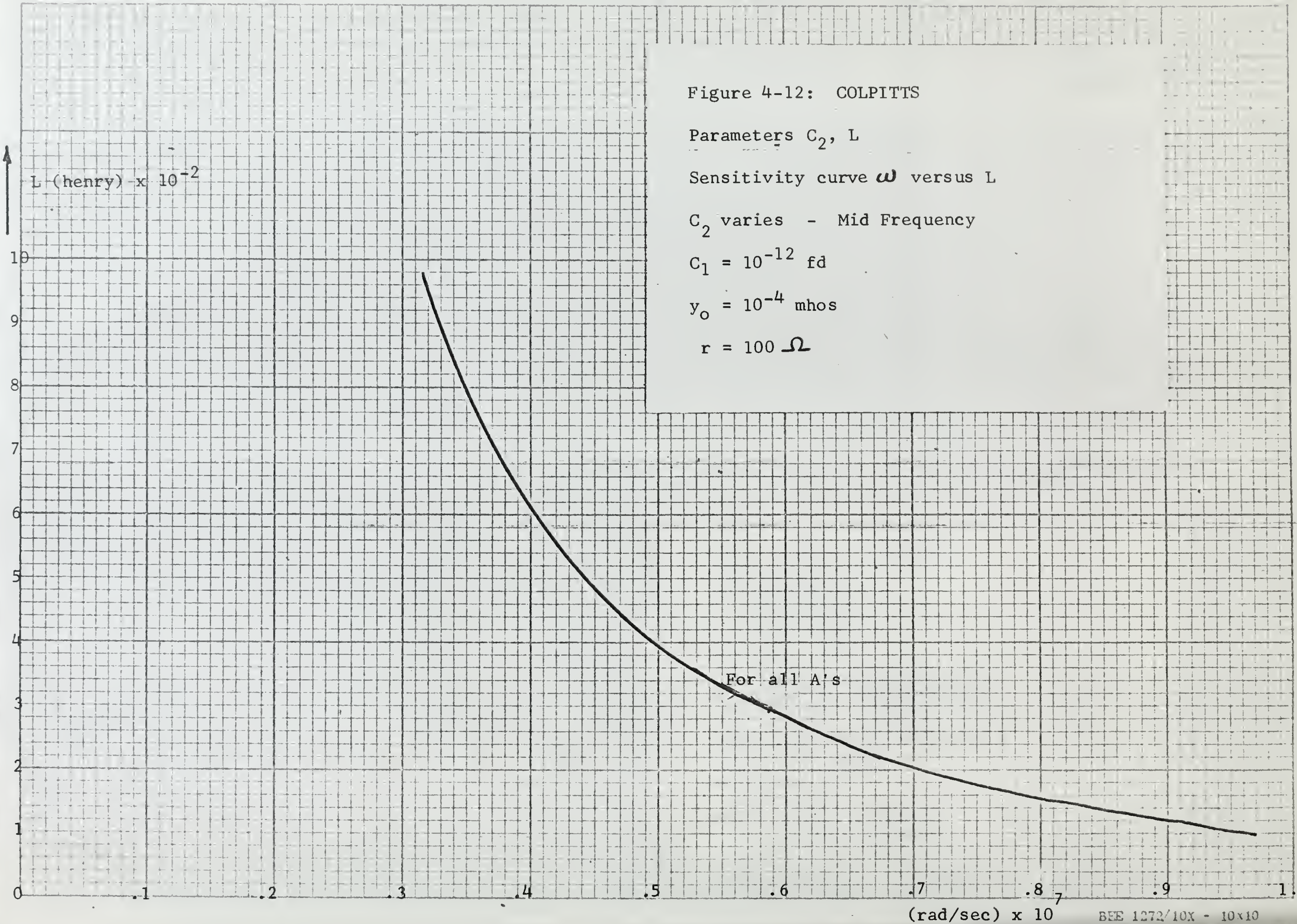


Figure 4-12: COLPITTS

Parameters C_2, L

Sensitivity curve ω versus L

C_2 varies - Mid Frequency

$C_1 = 10^{-12}$ fd

$y_o = 10^{-4}$ mhos

$r = 100\ \Omega$

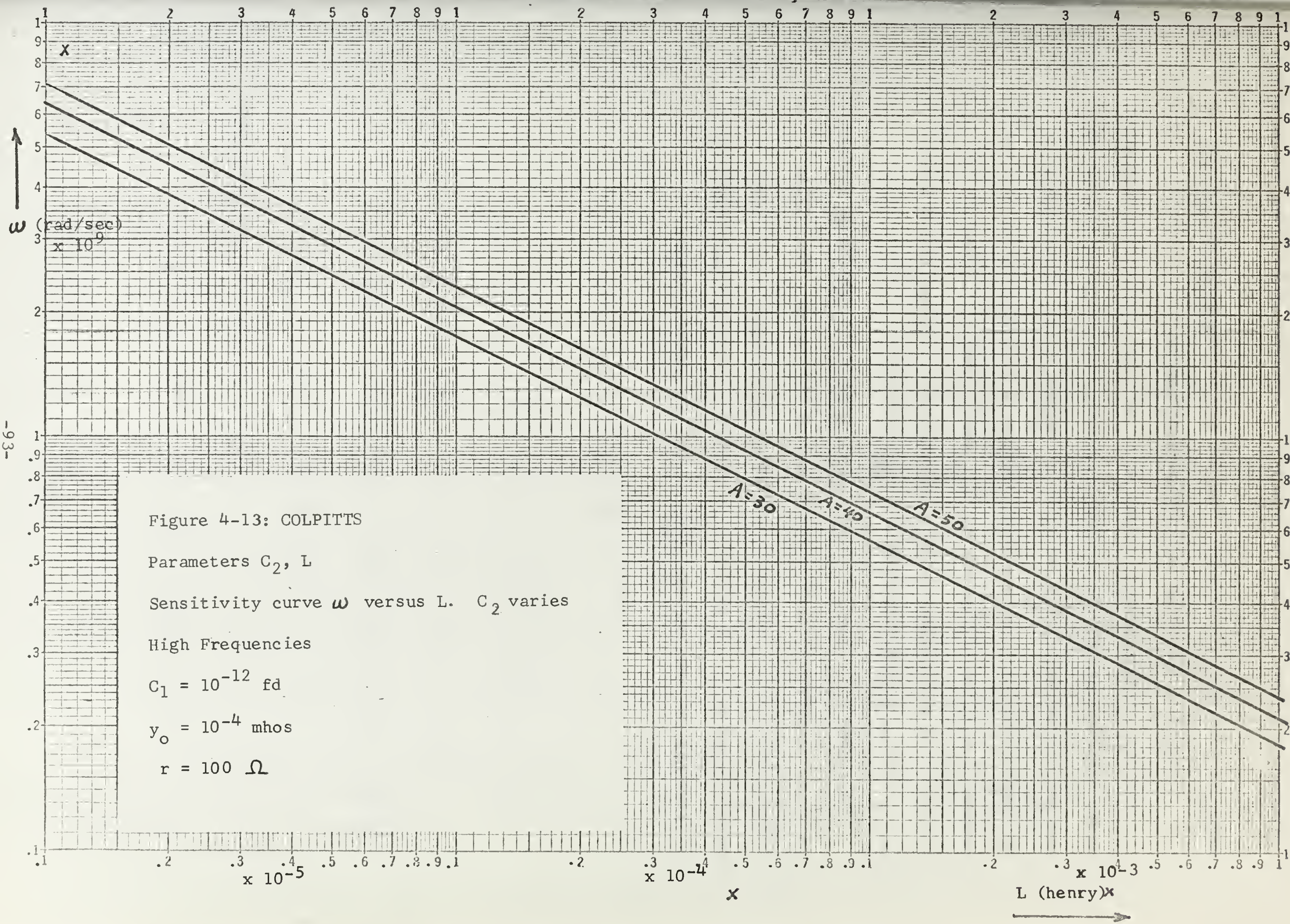


Figure 4-13: COLPITTS

Parameters C_2, L

Sensitivity curve ω versus L. C_2 varies

High Frequencies

$C_1 = 10^{-12}$ fd

$y_o = 10^{-4}$ mhos

$r = 100\ \Omega$

meter L), i.e. for $C_2 = .2 \times 10^{-13}$ fd for A = 50 (Figure 4-10).

4-7. Sensitivity curves ω versus L with C_2 varying.

For low and middle frequencies, up to $\omega = 10^7$ rad/sec the frequency curves are the same for all A's (Figures 4-11, 4-12). If we work on the left part of the curve, say $\omega < .5 \times 10^7$ (Figure 4-12), for a change in the lower values of L we have wider changes in frequency rather than for changes of L in the higher values, i.e. for a change of L from 8×10^{-2} h to 7×10^{-2} h we have $\Delta\omega = 0.225 \times 10^6$ while for a change from 5×10^{-2} to 4×10^{-2} we have $\Delta\omega = .5 \times 10^6$ rad/sec.

If L goes to a lower and lower value we have a wider change in frequency as we approach the flattening part of the curves. For higher frequencies ($\omega \doteq 10^9$ rad/sec) we have different curves for different gains (Figure 4-13) and variation of frequency with L is logarithmic and it is always the same because $\frac{d\omega}{\omega} / \frac{dL}{L}$ is almost constant.

5. Hartley Oscillator.

5-1. Derivation of characteristic equation.

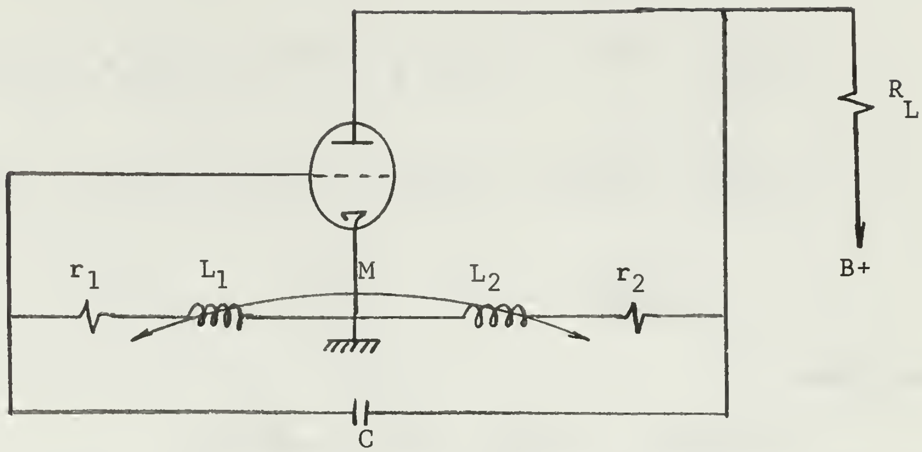


Figure 5-1. Hartley Oscillator

Characteristic equation:

$$y_f Z_f + y_o Z_i + 1 = 0 \qquad y_f = g_m$$

$$y_o = \frac{1}{r_p // R_L} = \frac{r_p + R_L}{r_p R_L}$$

$$Z_f = \frac{1}{Z}\left[r_1 r_2 - \frac{M}{C} - \omega^2 L_1 L_2 + \omega^2 M^2 \right] + \frac{j}{Z}(L_1 r_2 + L_2 r_1)$$

$$= \frac{1}{Z}\left[r_1 r_2 - \frac{M}{C} + S^2 L_1 L_2 - S^2 M^2 + SL_1 r_2 + SL_2 r_1 \right]$$

$$Z_i = \frac{1}{Z}\left[r_1 r_2 + \frac{L_2}{C} + S^2 L_1 L_2 - S^2 M^2 + SL_1 r_2 + SL_2 r_1 + \frac{r_2}{SC} \right]$$

$$Z = r_1 + r_2 + SL_1 + SL_2 + 2SM + \frac{1}{SC}$$

Plug into the characteristic equation:

$$g_m \left[\frac{1}{Z}(r_1 r_2 - \frac{M}{C} + S^2 L_1 L_2 - S^2 M^2 + SL_1 r_2 + SL_2 r_1) \right]$$

$$+ y_o \left[\frac{1}{Z}(r_1 r_2 + \frac{L_2}{C} + S^2 L_1 L_1 - S^2 M^2 + S L_1 r_2 + S L_2 r_1 \right.$$

$$\left. + \frac{r_2}{SC}) \right] + 1 = 0$$

$$g_m r_1 r_2 - \frac{g_m M}{C} + S^2 g_m L_1 L_2 - S^2 M^2 g_m + S L_1 r_2 g_m + S L_2 r_1 g_m +$$

$$y_o r_1 r_2 + \frac{L_2 y_o}{C} + S^2 L_1 L_2 y_o - S^2 M^2 y_o + S L_1 r_2 y_o + S L_2 r_1 y_o$$

$$+ \frac{r_2 y_o}{SC} + Z = 0$$

which gives

$$S^3 (C g_m L_1 L_2 - C M^2 g_m + C L_1 L_2 y_o - C M^2 y_o) +$$

$$S^2 (C L_1 r_2 g_m + C L_2 r_1 g_m + C L_1 r_2 y_o + C L_2 r_1 y_o + C L_1 + C L_2 + 2CM) +$$

$$S (C g_m r_1 r_2 - g_m M + C y_o r_1 r_2 + L_2 y_o + C r_1 + C r_2) + (r_2 y_o + 1) = 0$$

Divide by y_o and set $A = \frac{g_m}{y_o}$ and we get

$$S^3 (A C L_1 L_2 - A C M^2 + C L_1 L_2 - C M^2)$$

$$+ S^2 (A C L_1 r_2 + A C L_2 r_1 + C L_1 r_2 + C L_2 r_1 + \frac{C L_1 + C L_2 + 2CM}{y_o})$$

$$+ S (A C r_1 r_2 - A M + C r_1 r_2 + L_2 + \frac{C r_1 + C r_2}{y_o} + (r_2 + \frac{1}{y_o}) = 0$$

5-2. Parameter plane curves C versus L_1 for different gains.

Giving characteristic values to the tube circuit parameters such as:

$y_o = 10^{-4}$ mhos

$r_1 = r_2 = 100\,\Omega$

$M = 10^{-4}$ henries

$L_2 = 10^{-3}$ henries

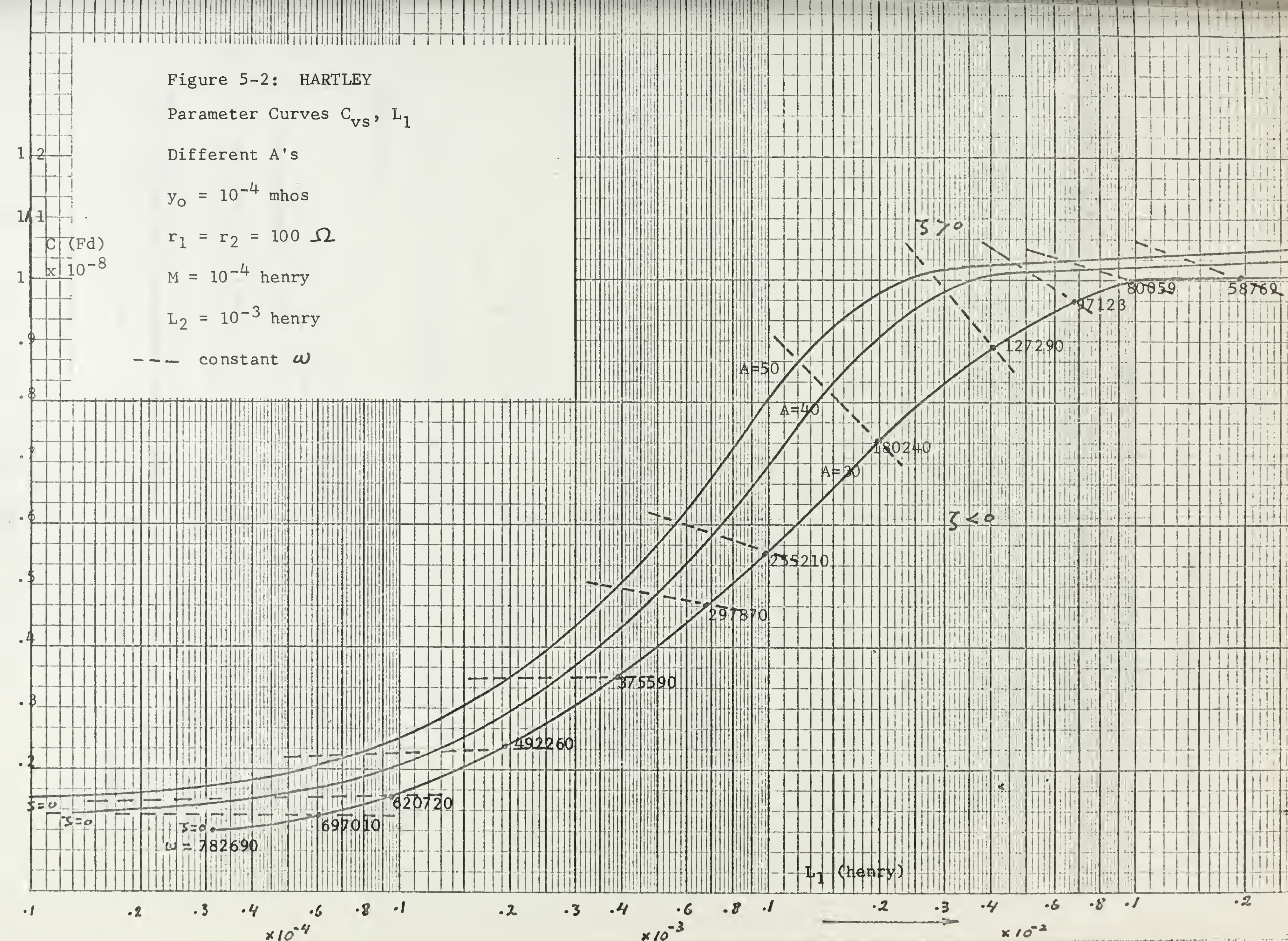


Figure 5-2: HARTLEY

Parameter Curves C_{vs}, L_1

Different A's

$y_o = 10^{-4}$ mhos

$r_1 = r_2 = 100\ \Omega$

$M = 10^{-4}$ henry

$L_2 = 10^{-3}$ henry

--- constant ω

Characteristic equation becomes:

$$S^3\left[CL_1(AL_2 + L_2) - C(AM^2 + M^2)\right] +$$

$$S^2\left[CL_1(Ar_2 + r_2 + \frac{1}{y_o}) + C(AL_2r_1 + L_2r_1 + \frac{L_2 + 2M}{y_o}\right] +$$

$$S\left[C(Ar_1r_2 + r_1r_2 + \frac{r_1 + r_2}{y_o}) + (L_2 - AM)\right] + \left[r_2 + \frac{1}{y_o}\right] = 0$$

$$S^3\left[CL_1(0.31 \times 10^{-1}) - C(0.31 \times 10^{-6})\right] +$$

$$S^2\left[CL_1(1.31 \times 10^4) + C(15.1)\right] + S\left[C(2.3 \times 10^6) - 2 \times 10^{-3}\right]$$

$$+ 1.01 \times 10^4 = 0$$

<u>For A = 40</u>

$$S^3\left[CL_1(0.41 \times 10^{-1}) - C(0.41 \times 10^{-6})\right] +$$

$$S^2\left[CL_1(1.41 \times 10^4) + C(16.1)\right] + S\left[C(2.41 \times 10^6) - 3 \times 10^{-3}\right]$$

$$+ 1.01 \times 10^4 = 0$$

<u>For A = 50</u>

$$S^3\left[CL_1(0.51 \times 10^{-1}) - C(0.51 \times 10^{-6})\right] +$$

$$S^2\left[CL_1(1.51 \times 10^4) + C(17.1)\right] + S\left[C(2.5 \times 10^6) - 4 \times 10^{-3}\right]$$

$$+ 1.01 \times 10^4 = 0$$

<u>Results for Hartley oscillator: Parameters C, L_1</u>

For low frequencies up to $\omega \doteq 90$ krad/sec C remains constant and its value is higher for higher gains. For A = 30, $C \doteq 1.01 \times 10^{-8}$ fd and for A = 50, $C \doteq 1.07 \times 10^{-8}$ fd.

For high frequencies C also remains almost constant (for $>$ 800 krad/sec).

In the case of the mid frequencies, in order to achieve a certain frequency of oscillation we must vary both variables C and L_1 (Figure 5-2).

5-3. Parameter plane curves C versus L_2 for different gains.

Giving typical values to the tube circuit parameters as:

$y_o = 10^{-4}$ mhos

$r_1 = r_2 = 100\ \Omega$

$M = 10^{-4}$ henry

$L_1 = 10^{-3}$ henry

Characteristic equation:

$$s^3\left[CL_2(AL_1 + L_1) - C(AM^2 + M^2)\right] +$$

$$s^2\left[CL_2(Ar_1 + r_1 + \frac{1}{y_o}) + C(AL_1r_2 + L_1R_2 + \frac{L_1 + 2M}{y_o}\right] +$$

$$s\left[C(Ar_1r_2 + r_1r_2 + \frac{r_1 + r_2}{y_o}) + L_2 - AM\right] + \left[r_2 + \frac{1}{y_o}\right] = 0$$

<u>For A = 30</u>

$$s^3\left[CL_2(0.31 \times 10^{-1}) - C(0.31 \times 10^{-6})\right] +$$

$$s^2\left[CL_2(0.131 \times 10^5) + C(15.1)\right] + s\left[C(2.31 \times 10^6) + L_2 - 0.3 \times 10^{-2}\right] + 1.01 \times 10^4 = 0$$

<u>For A = 40</u>

$$s^3\left[CL_2(0.41 \times 10^{-1}) - C(0.41 \times 10^{-6})\right] +$$

$$s^2\left[CL_2(0.141 \times 10^5) + C(16.1)\right] + s\left[C(2.41 \times 10^6) + L_2 - 0.4 \times 10^{-2}\right. + 1.01 \times 10^4 = 0$$

We also get similar equations for A = 70 and A = 90.

<u>Results for Hartley: Parameters C, L_2</u>

There are 2 branches of curves $\zeta = 0$ and they leave a gap between

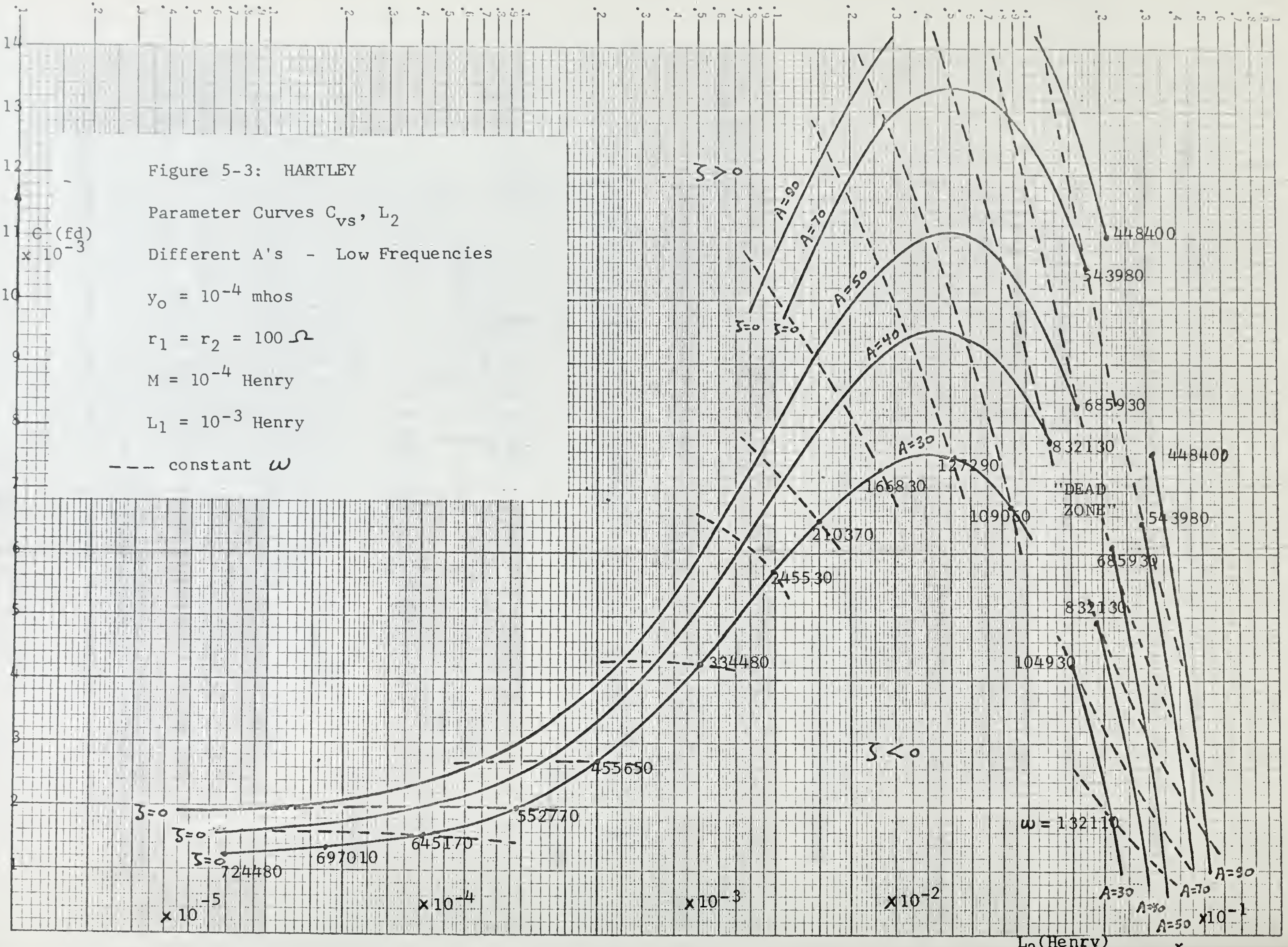


Figure 5-3: HARTLEY

Parameter Curves C_{vs}, L_2

Different A's - Low Frequencies

$y_o = 10^{-4}$ mhos

$r_1 = r_2 = 100\,\Omega$

$M = 10^{-4}$ Henry

$L_1 = 10^{-3}$ Henry

--- constant ω

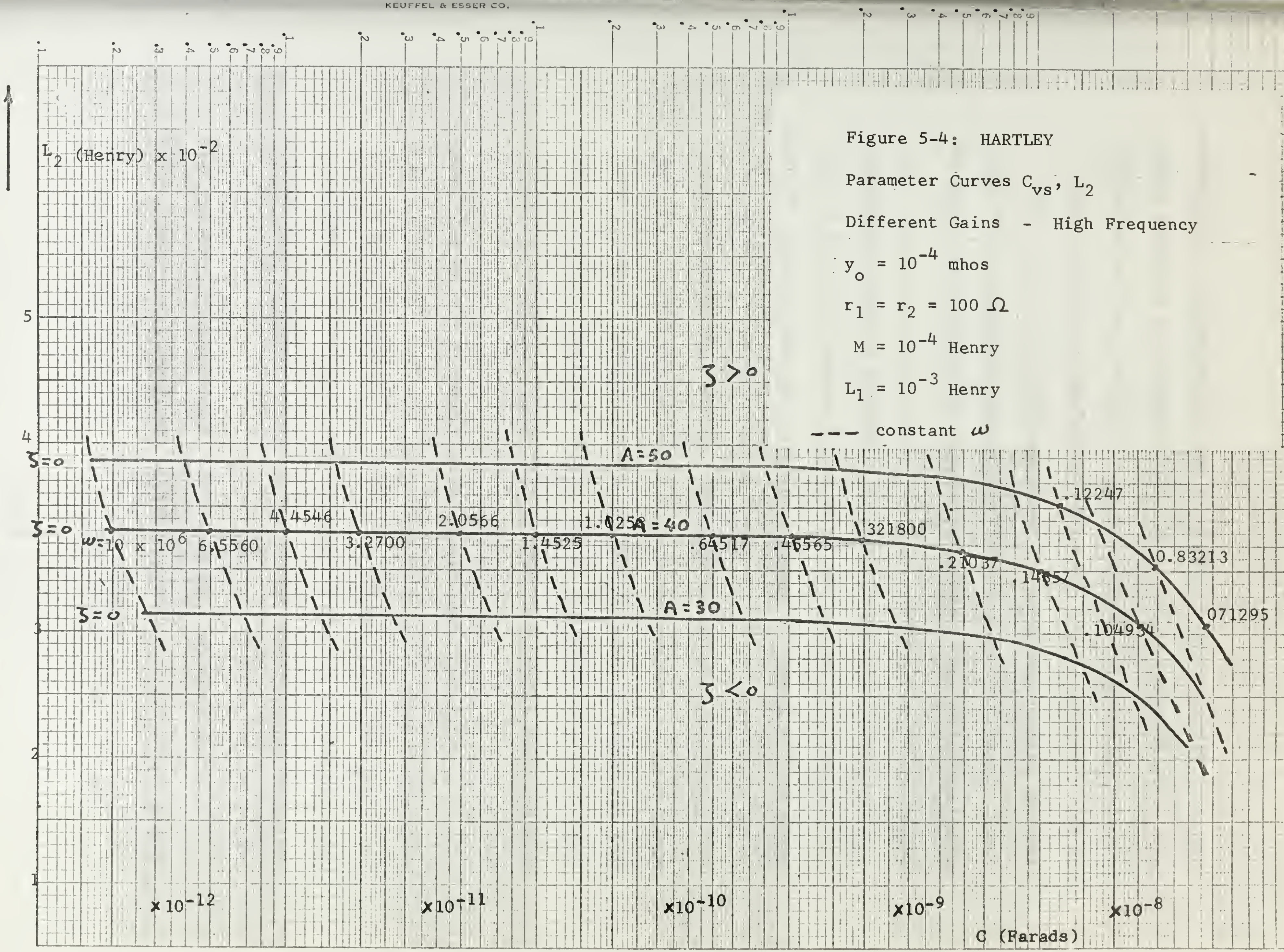


Figure 5-4: HARTLEY

Parameter Curves C_{vs}, L_2

Different Gains - High Frequency

$y_o = 10^{-4}$ mhos

$r_1 = r_2 = 100\ \Omega$

$M = 10^{-4}$ Henry

$L_1 = 10^{-3}$ Henry

--- constant ω

them where there are complex values of variable parameters. For example, for A = 40, frequency starts at ω = 83213 rad/sec and increases to the left, for C = 7.7 x 10^{-9} farads and L_2 = 0.0125 h. Also we can achieve the same frequency for C = 4.9 x 10^{-9} farads and L_2 = 0.0195 h, and from that point on, frequency increases down the curve on the other branch. So we see that we can achieve the same frequency for two different sets of values of parameters, one at one branch of the curve and another at the other branch.

The constant frequency curves are consistent for both branches of ζ = 0 curves. In other words if we extend constant ω curves of one branch of ζ = 0 curves, they fall on top of the same constant ω curves of the other branch of ζ = 0 curves as the constant ω = 104930 rad/sec.

If we follow one branch of curves, say the right hand part of Figure 5-3, it continues on Figure 5-4 for higher frequencies, we see that for higher frequencies such as $\omega >$ 800 krad/sec L_2 remains constant, $L_2 \doteq$ 38.0 mh for A = 50 and is lower for lower gains, L_2 = 26 mh for A = 30. So in this area of curves we have frequency control by varying only C.

If we follow the other branch of the curves ζ = 0, we can increase frequency of oscillation by decreasing L_2, and first increasing, then decreasing and lastly keeping C constant. For frequencies up to ω = 93442 rad/sec for A = 50, C increases then up to $\omega \doteq$ 700 krad/sec, C decreases and for higher frequencies C remains constant, while the frequency is controlled by varying only L_2.

In other words for high frequencies $\omega >$ 700 krad/sec, we can have either capacitive or inductive control. For lower frequencies ($\omega <$ 700 krad/sec) we control frequency by varying both parameters.

There is a maximum and minimum value of C for a given gain. For example, for A = 40, C varies from a minimum value of C = 1.6 μf to a maximum of C = 9.5 μf. These values go higher for higher gains, as for A = 50, with minimum C = 1.9 μf and maximum C = 11.1 μf.

Also there is a maximum value of L_2, such as $L_2 = 0.026$ h for A = 30 which goes higher for higher gains, $L_2 = 0.038$ h for A = 50.

There is a "dead zone" between the 2 branches of $\zeta = 0$ curves, where it seems that the oscillator does not oscillate, because the parameters have complex values. Let us test one of the points of the "dead zone", say $L_2 = 0.2 \times 10^{-1}$ h and $C = 7 \times 10^{-9}$ farads and for gain A = 50. We plug those values into the characteristic equation:

$$S^3 \left[(7 \times 10^{-9})(0.2 \times 10^{-1})(0.51 \times 10^{-1}) - 7 \times 10^{-9} (0.51 \times 10^{-6})\right] +$$
$$S^2 \left[(7 \times 10^{-9})(0.2 \times 10^{-1})(0.151 \times 10^{5}) + 7 \times 10^{-9}(17.1)\right] +$$
$$S \left[(7 \times 10^{-9})(2.51 \times 10^{6}) + 0.3 \times 10^{-1} - 0.5 \times 10^{-2}\right] +$$
$$1.01 \times 10^{4} = 0$$

Set $S = j\omega$ and equalizing real and imaginary parts to zero we get for the real part:

$$-\omega^2 \left[0.212 \times 10^{-5} + 0.12 \times 10^{-7}\right] = -1.01 \times 10^{4}$$

$$\omega^2 = \frac{1.01 \times 10^{4}}{0.213 \times 10^{-5}} = 4.75 \times 10^{9}$$

$\omega = 6.85 \times 10^{4}$ rad/sec which is almost the same as the constant curve $\omega = 6.8593 \times 10^{4}$ which falls on the selected point. So apparently the system must oscillate at this frequency.

5-4. Parameter plane curves C versus A.

Giving typical values to tube circuit parameters as follows:

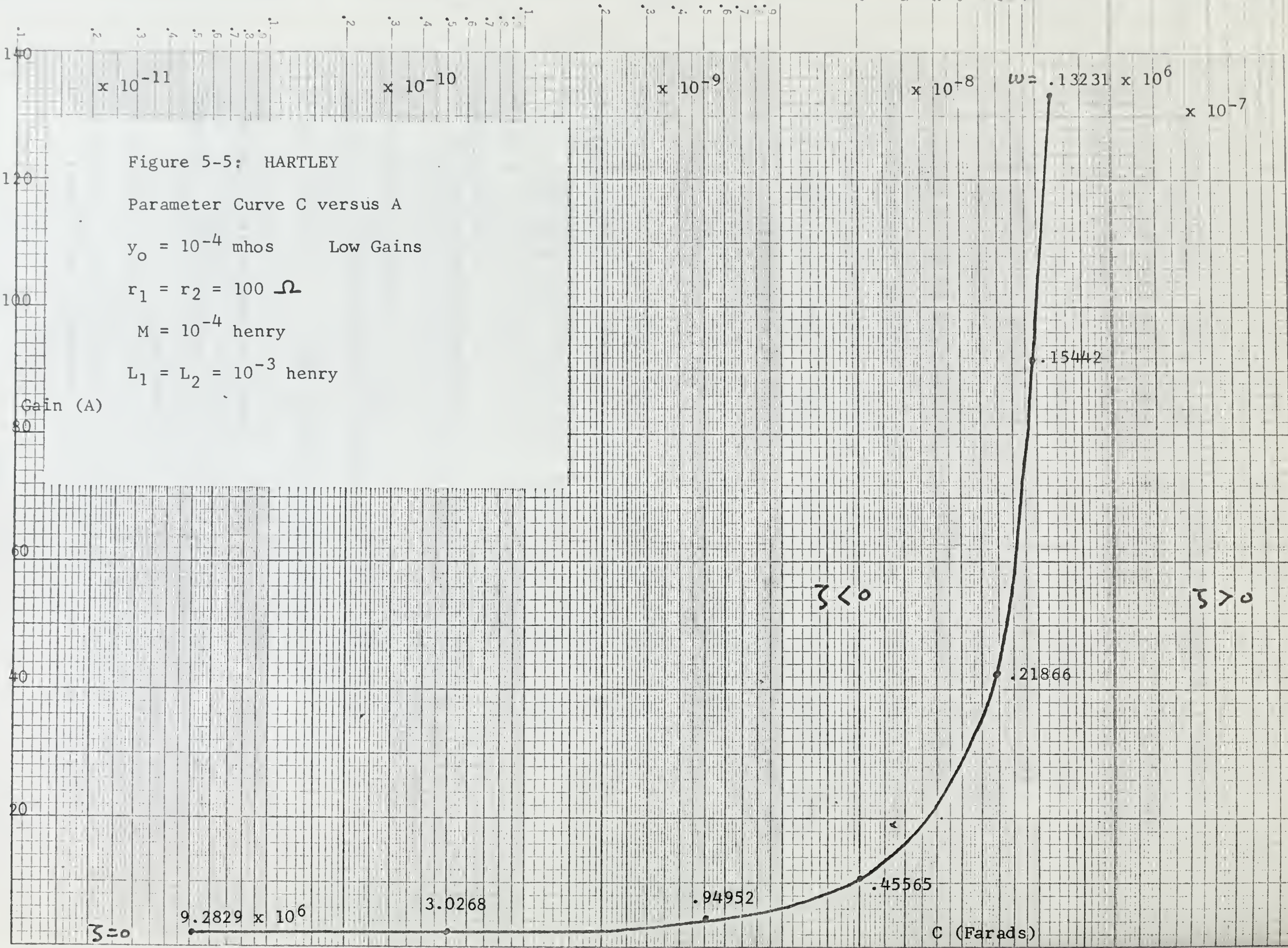


Figure 5-5: HARTLEY

Parameter Curve C versus A

$y_o = 10^{-4}$ mhos Low Gains

$r_1 = r_2 = 100\ \Omega$

$M = 10^{-4}$ henry

$L_1 = L_2 = 10^{-3}$ henry

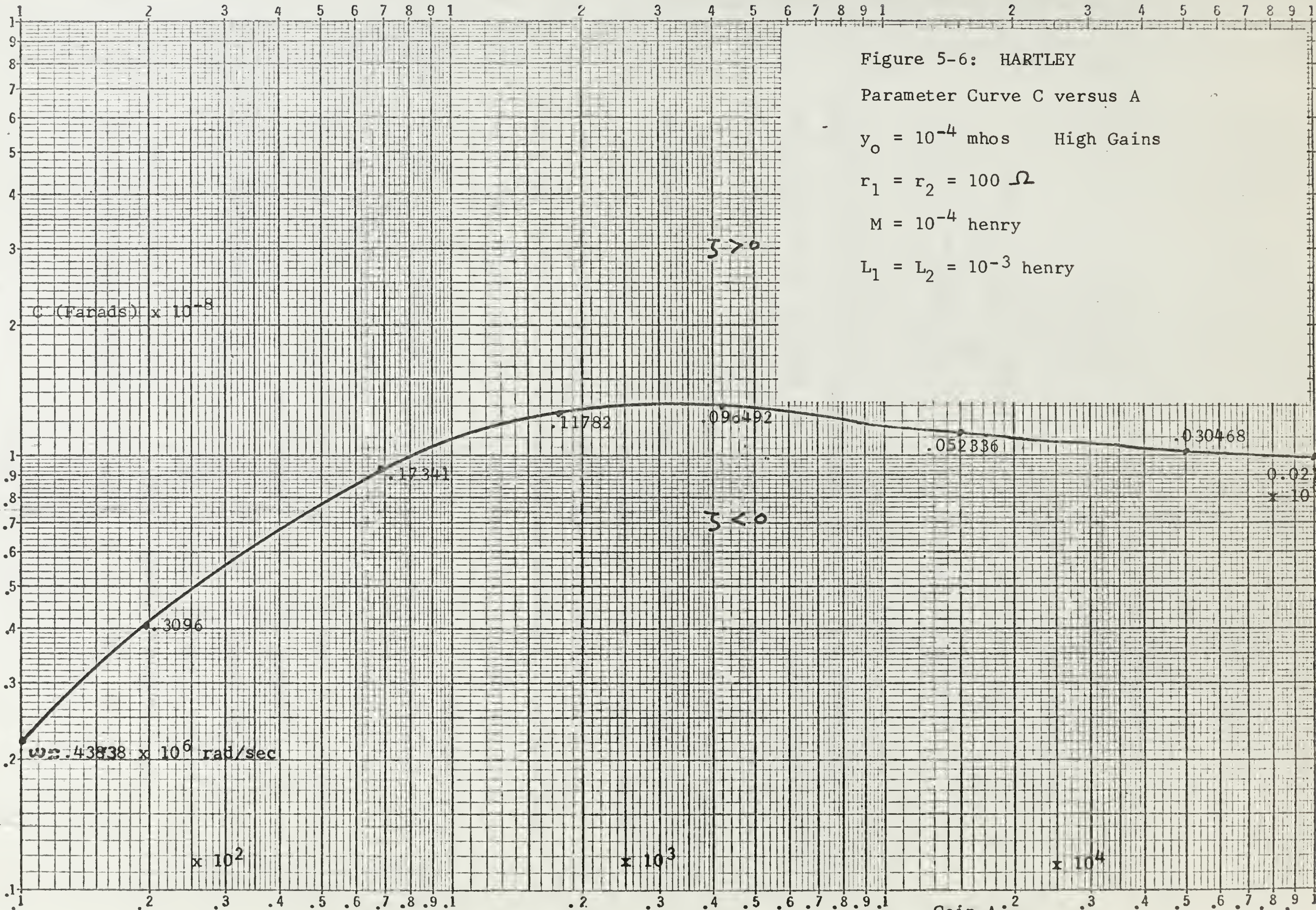


Figure 5-6: HARTLEY

Parameter Curve C versus A

$y_o = 10^{-4}$ mhos High Gains

$r_1 = r_2 = 100\ \Omega$

$M = 10^{-4}$ henry

$L_1 = L_2 = 10^{-3}$ henry

$y_o = 10^{-4}$ mhos

$r_1 = r_2 = 100\,\Omega$

$M = 10^{-4}$ henry

$L_1 = L_2 = 10^{-3}$ henry

Characteristic equation becomes:

$$S^3\left[AC(.99 \times 10^{-6}) + C(.99 \times 10^{-6})\right] + S^2\left[AC(2 \times 10^{-1}) + C(22.2)\right] + S\left[AC(10^4) + C(.201 \times 10^7) - A(10^{-4}) + 10^{-3}\right] + 1.01 \times 10^4 = 0$$

From Figure 5-5, for high frequencies, we observe that in the part of the high frequencies of the curve, for $\omega > 1.5 \times 10^6$ rad/sec, the variation of frequency is independent of gain A which reaches its minimum value A=1 for this range of frequencies.

For the rest of the curve both gain and capacitor change is required in order to get frequency variation.

For lower frequencies $\omega < .17 \times 10^6$ rad/sec, Figure 5-6, We see that capacitor C reaches a maximum value $C = 1.3 \times 10^{-8}$ fd for a gain of about A = 300 and then drops to a limit value $C = 1 \times 10^{-8}$ fd for very high gains.

5-5. Sensitivity curves ω versus C with L varying.

For frequencies less than 6×10^4 rad/sec and down to $.8 \times 10^4$ the parameter C is constant for each value of gain, i.e. $C = 1.01 \times 10^{-5}$ fd for A = 30 (Figure 5-7). The frequency is controlled only by means of the other parameter L_1. In other words, if L_1 is inadvertently varied then we have frequency instability.

For frequencies greater than 6×10^4 rad/sec, frequency varies with C and the lower C is the larger the frequency variation is, i.e.

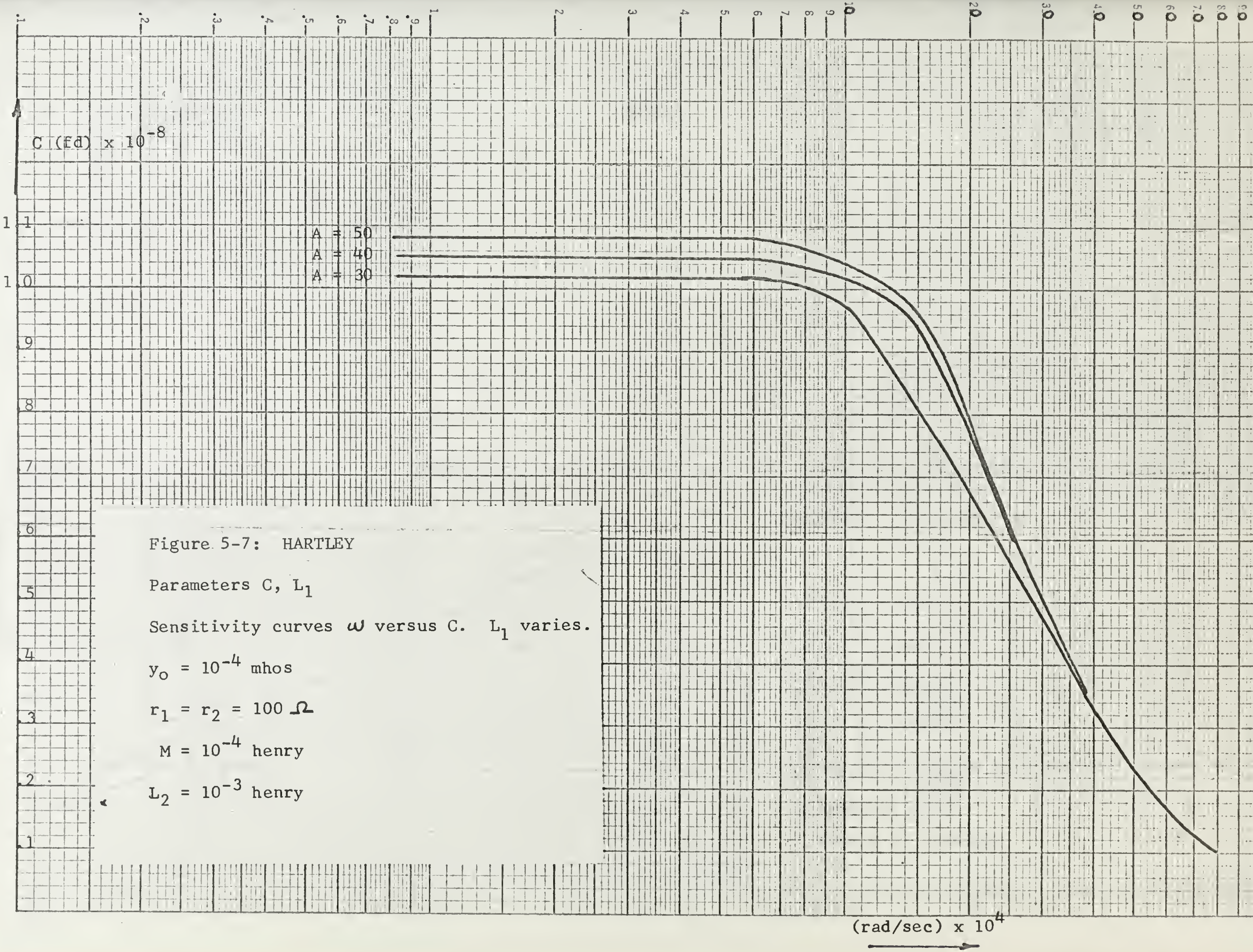


Figure 5-7: HARTLEY

Parameters C, L_1

Sensitivity curves ω versus C. L_1 varies.

$y_o = 10^{-4}$ mhos

$r_1 = r_2 = 100\ \Omega$

$M = 10^{-4}$ henry

$L_2 = 10^{-3}$ henry

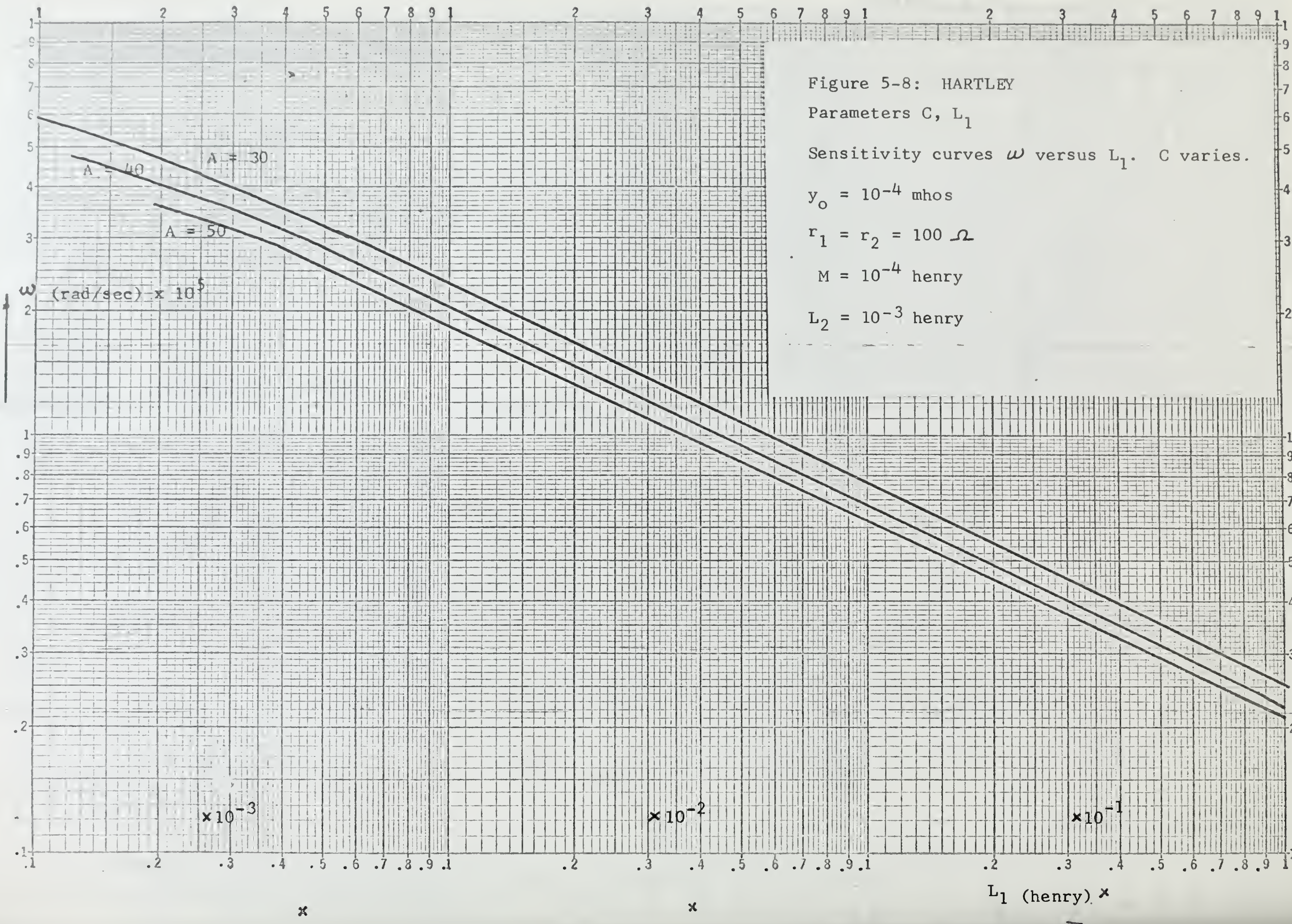


Figure 5-8: HARTLEY

Parameters C, L_1

Sensitivity curves ω versus L_1. C varies.

$y_o = 10^{-4}$ mhos

$r_1 = r_2 = 100\ \Omega$

$M = 10^{-4}$ henry

$L_2 = 10^{-3}$ henry

for A = 30 (Figure 5-7) for a change of C from $.7 \times 10^{-8}$ to $.6 \times 10^{-8}$ f, $\Delta\omega = 4 \times 10^{4}$ rad/sec while for a variation of C from $.3 \times 10^{-8}$ to $.2 \times 10^{-8}$ fd $\Delta\omega = 12 \times 10^{4}$ rad/sec.

For higher frequencies ($\omega > 4 \times 10^{5}$ rad/sec) the curves for different gains coincide so frequency curves are independent of gain.

5-6. Sensitivity curves ω versus L_1 with C varying.

The curves for different A's have the same slope so $\frac{\Delta\omega}{\omega} / \frac{\Delta L_1}{L_1}$ is almost constant (Figure 5-8).

5-7. Sensitivity curves ω versus C with L_2 varying.

For low frequencies, from 1×10^{5} to 7×10^{5} rad/sec (Figure 5-9) one value of C can give 2 values of frequency, as for $C = 5 \times 10^{-9}$ fd and A = 50, $\omega = .8 \times 10^{5}$ rad/sec or $\omega = 3 \times 10^{5}$ rad/sec.

The constant gain curves fall on top of each other for $C < 3 \times 10^{-9}$ fd or frequency curves are independent of gain for $\omega > 4.5 \times 10^{5}$ rad/sec.

There is a gap between each constant A curve around $C = 6 \times 10^{-9}$ fd as it appears also on Figure 5-3.

For a given gain A there is a maximum value of C as for A = 40, maximum $C = 9.5 \times 10^{-9}$ fd which can produce frequency instability if the other parameter (L_2) is unstable.

For higher frequencies (Figure 5-10), the slope of the curve on log-log paper is almost constant so $\frac{\Delta\omega}{\omega} / \frac{\Delta C}{C}$ constant

5-8. Sensitivity curves ω versus L_2 with C varying.

There are 2 values of L_2 that can give the same frequency for a given gain A. (Figure 5-11). For A = 30, $\omega = 1 \times 10^{5}$ rad/sec for $L_2 = .1 \times 10^{-1}$ or $.15 \times 10^{-1}$ h and we can conclude from the analog computer test that the 2 branches of curves are connected by an almost straight line.

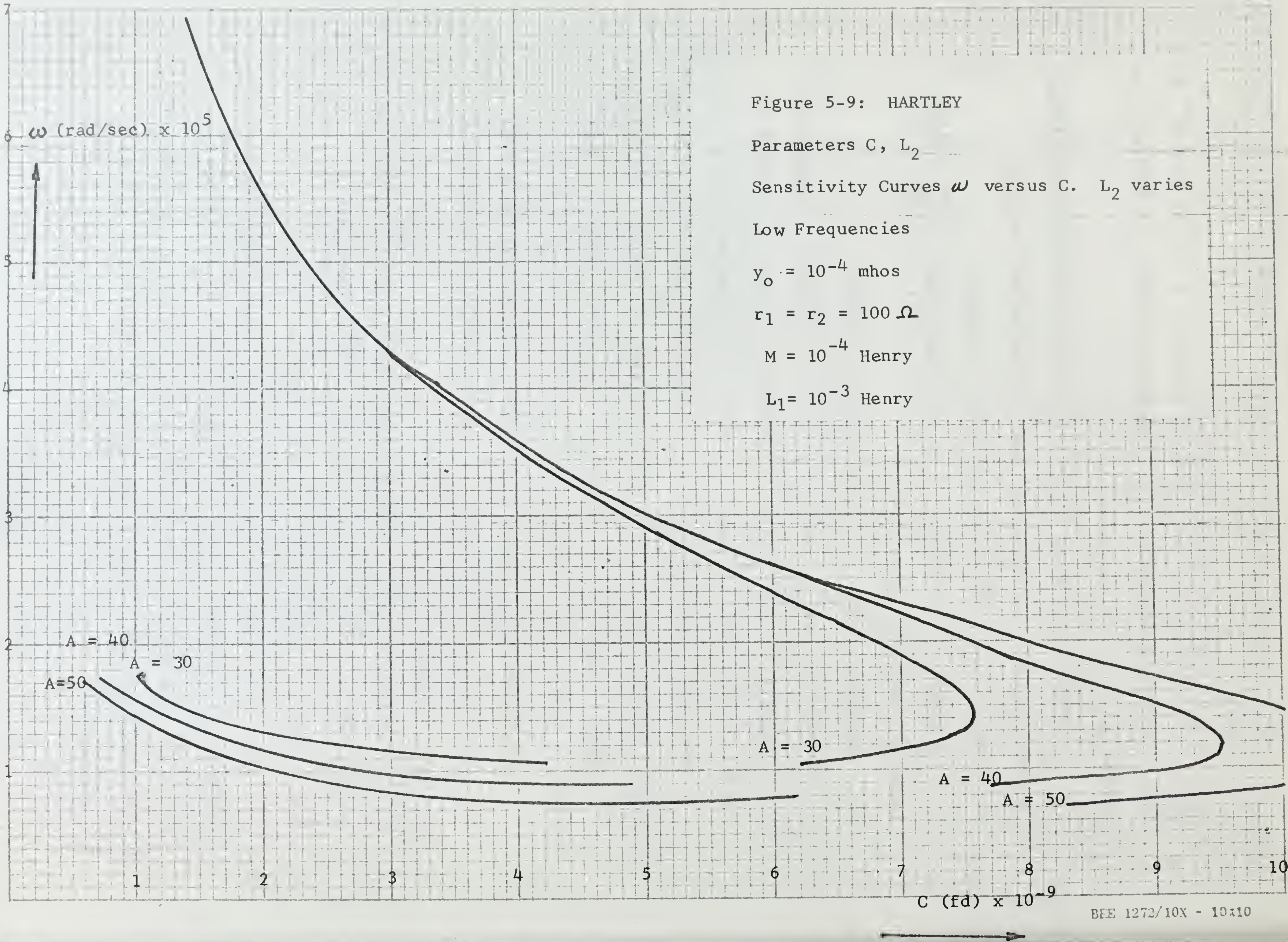


Figure 5-9: HARTLEY

Parameters C, L_2

Sensitivity Curves ω versus C. L_2 varies

Low Frequencies

$y_o = 10^{-4}$ mhos

$r_1 = r_2 = 100\ \Omega$

$M = 10^{-4}$ Henry

$L_1 = 10^{-3}$ Henry

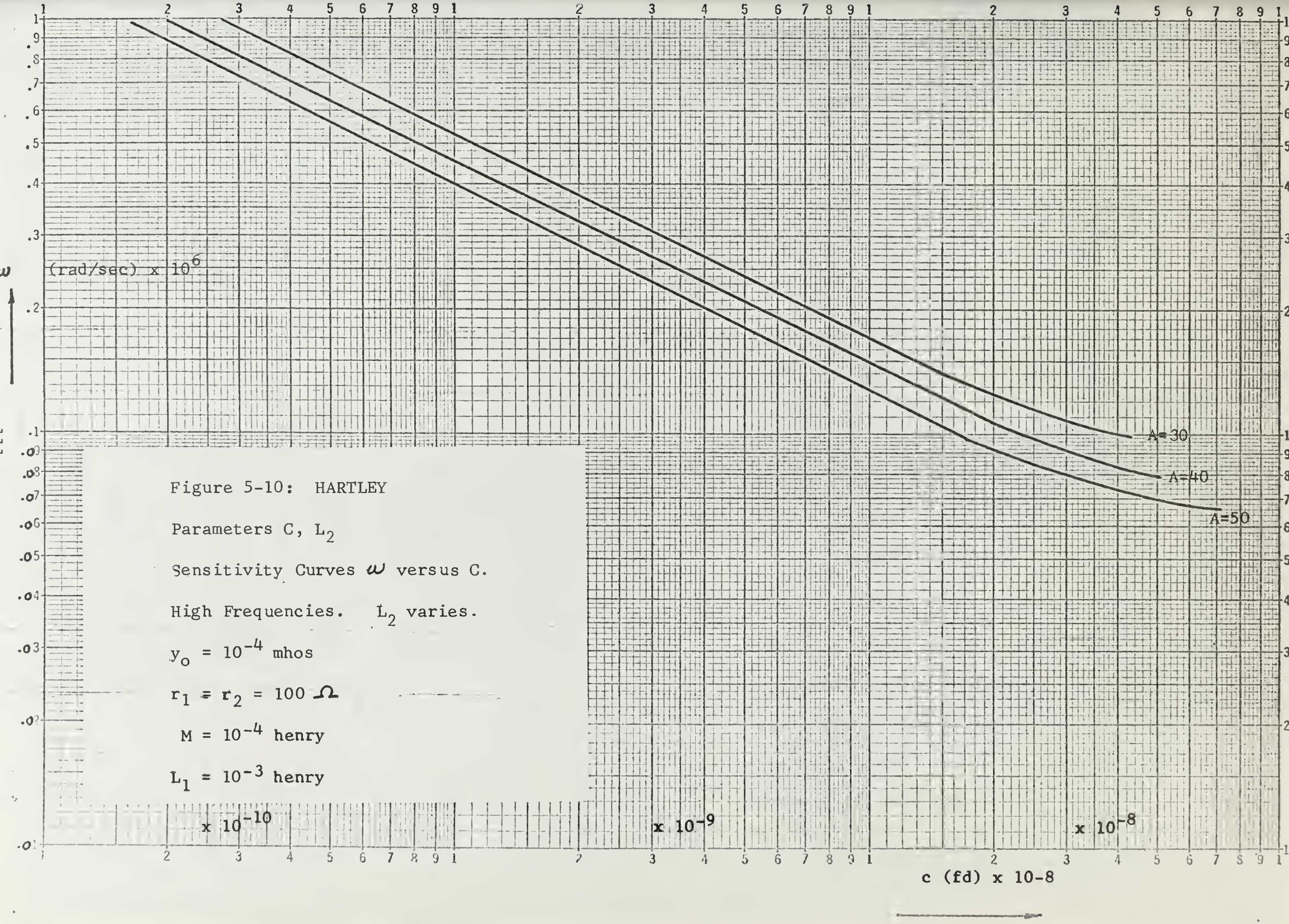


Figure 5-10: HARTLEY

Parameters C, L_2

Sensitivity Curves ω versus C.

High Frequencies. L_2 varies.

$y_o = 10^{-4}$ mhos

$r_1 = r_2 = 100\ \Omega$

$M = 10^{-4}$ henry

$L_1 = 10^{-3}$ henry

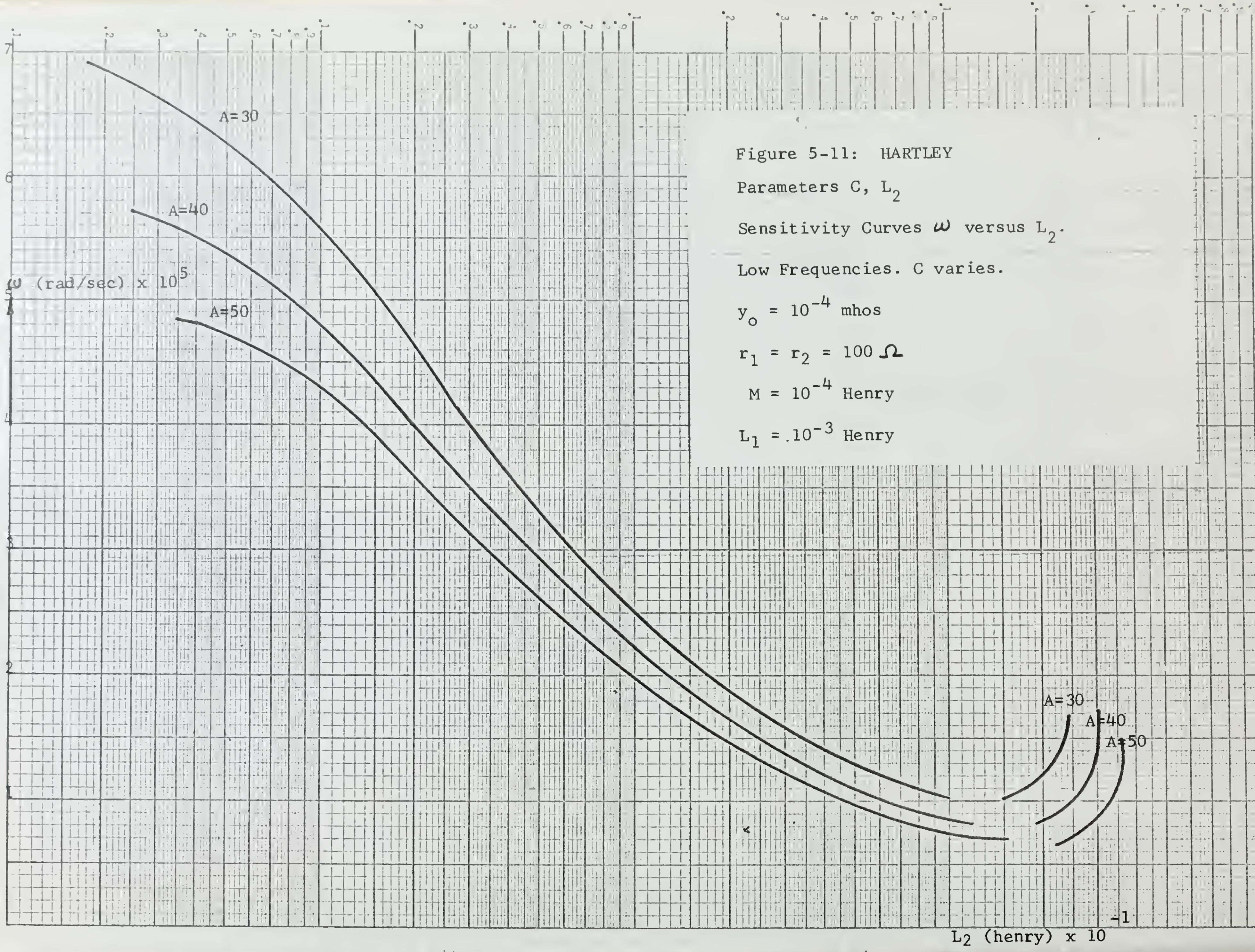


Figure 5-11: HARTLEY

Parameters C, L_2

Sensitivity Curves ω versus L_2.

Low Frequencies. C varies.

$y_o = 10^{-4}$ mhos

$r_1 = r_2 = 100\ \Omega$

$M = 10^{-4}$ Henry

$L_1 = .10^{-3}$ Henry

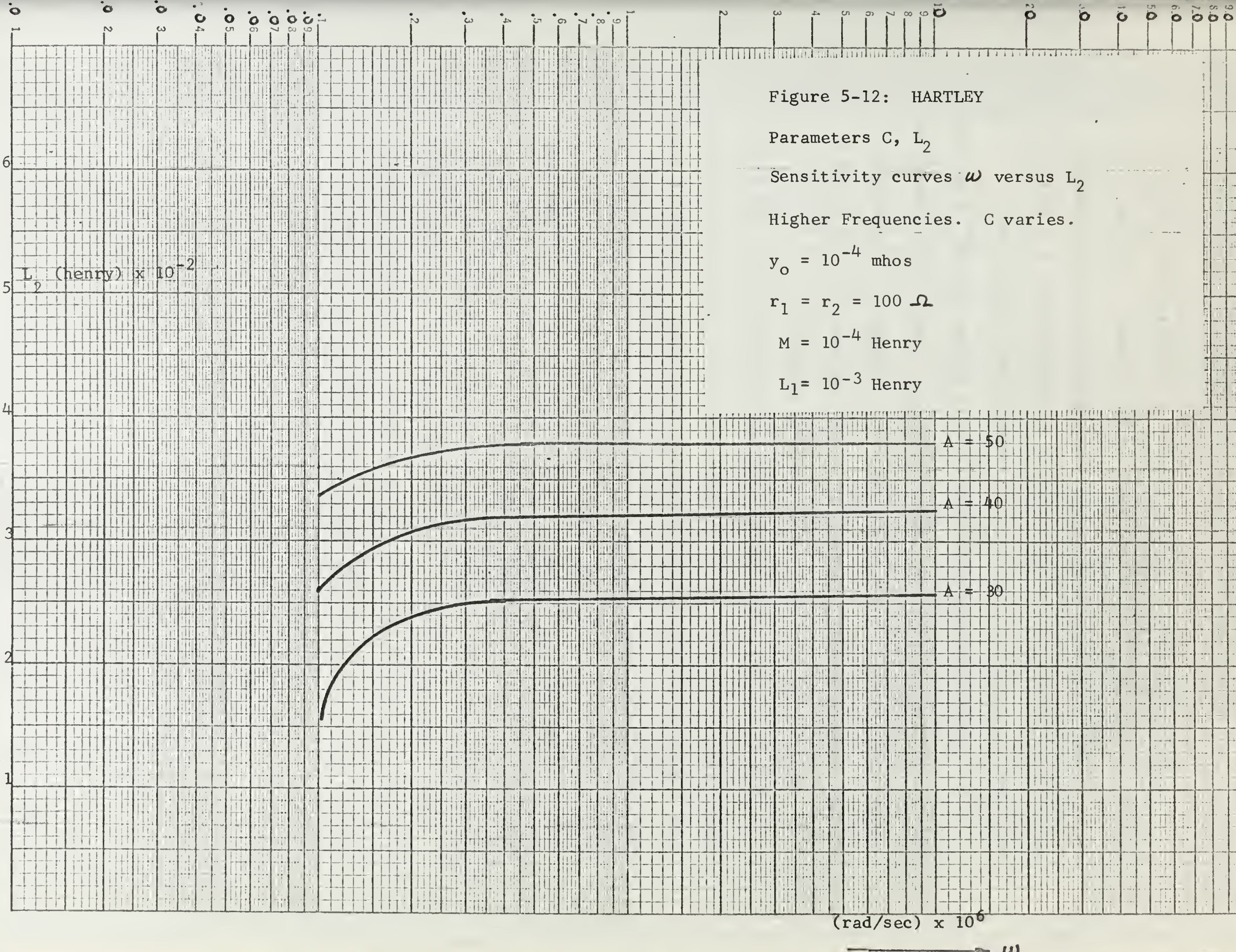


Figure 5-12: HARTLEY

Parameters C, L_2

Sensitivity curves ω versus L_2

Higher Frequencies. C varies.

$y_o = 10^{-4}$ mhos

$r_1 = r_2 = 100\ \Omega$

$M = 10^{-4}$ Henry

$L_1 = 10^{-3}$ Henry

There is a minimum value of frequency of oscillations, i.e. for $A = 30$, minimum $\omega = 1 \times 10^5$ rad/sec. As we decrease L_2 (Figure 5-11) we cause less and less change in ω as we approach the flattening part of the left branch. Also, for values of L_2 greater than $.5 \times 10^{-2}$ h, the left branch of curves flattens so we have less change in ω.

For higher frequencies and increasing L_2 beyond $.2 \times 10^{-1}$ h we pick up the right branch of curves which continues on Figure 5-12. There is a maximum value of L_2 for a given gain where frequency is varied only by the other parameter (C), i.e. for $A = 50$, maximum $L_2 = 3.8 \times 10^{-2}$ h.

5-9. Sensitivity curves ω versus A with C varying.

From figure 5-13 one observes that for high frequencies $\omega > 3$ Mrad/sec, A remains constant at its minimum value $A = 1$ while for $\omega < 3$ Mrad/sec, logarithmic variation of gain causes logarithmic variation of frequency (Figure 5-14).

5-10. Sensitivity curves ω versus C with A varying.

From the curve of Figure 5-15 for high frequencies $\omega > .2$Mrad/sec one observes that logarithmic variation of frequency corresponds to logarithmic variation of capacitor, while gain is changing accordingly.

For frequencies $\omega < 0.2$ Mrad/sec, (Figure 5-16), C reaches a peak value $C = 1.3 \times 10^{-8}$ fd at $\omega = .1$ Mrad/sec and then drops to a limiting value of $C = 1 \times 10^{-8}$ fd for lower frequencies $\omega < .02$ Mrad/sec.

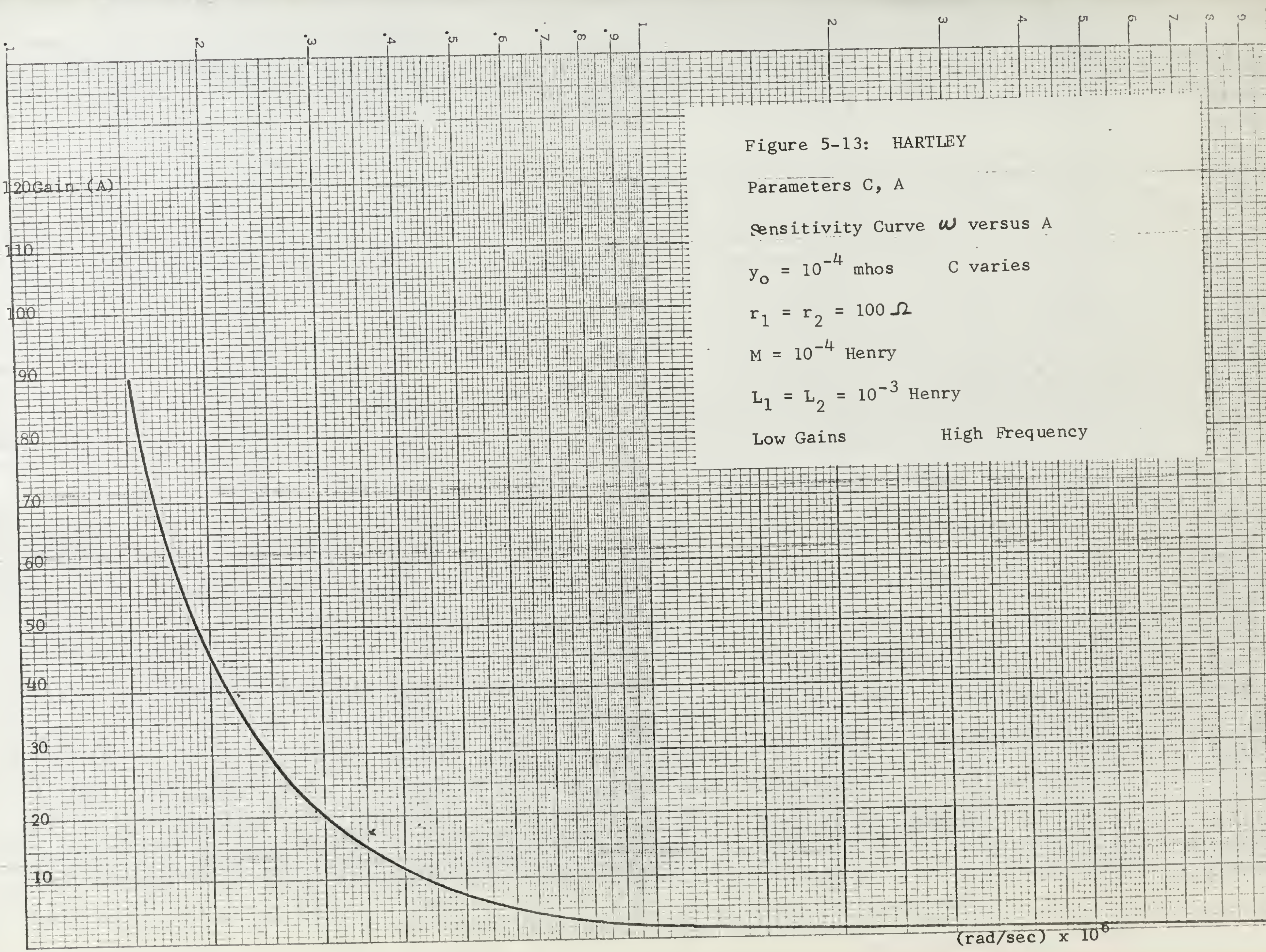


Figure 5-13: HARTLEY

Parameters C, A

Sensitivity Curve ω versus A

$y_o = 10^{-4}$ mhos C varies

$r_1 = r_2 = 100\,\Omega$

$M = 10^{-4}$ Henry

$L_1 = L_2 = 10^{-3}$ Henry

Low Gains High Frequency

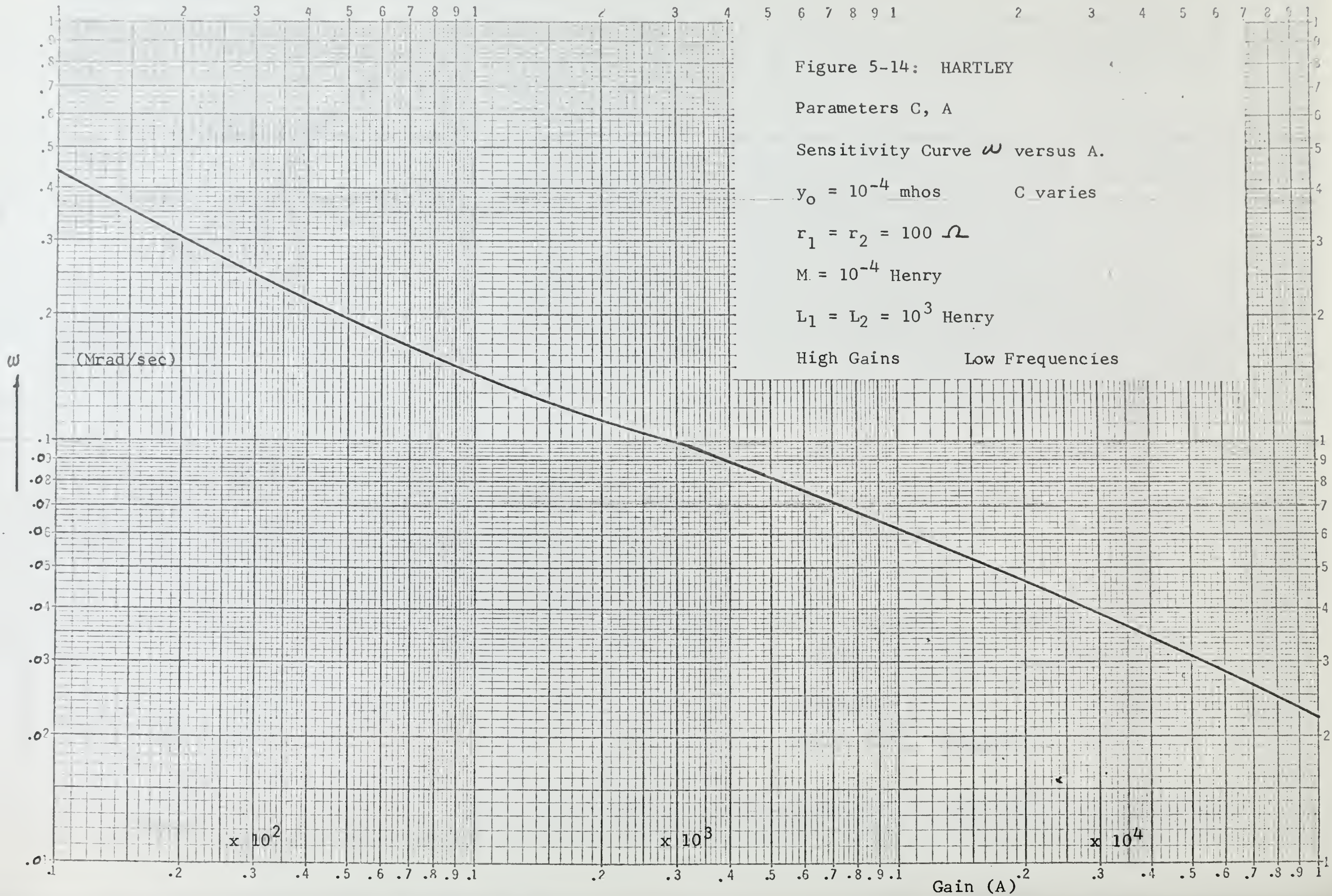


Figure 5-14: HARTLEY
Parameters C, A
Sensitivity Curve ω versus A.
$y_o = 10^{-4}$ mhos C varies
$r_1 = r_2 = 100\ \Omega$
$M = 10^{-4}$ Henry
$L_1 = L_2 = 10^3$ Henry
High Gains Low Frequencies
ω
(Mrad/sec)
x 10^2
x 10^3
x 10^4
Gain (A)

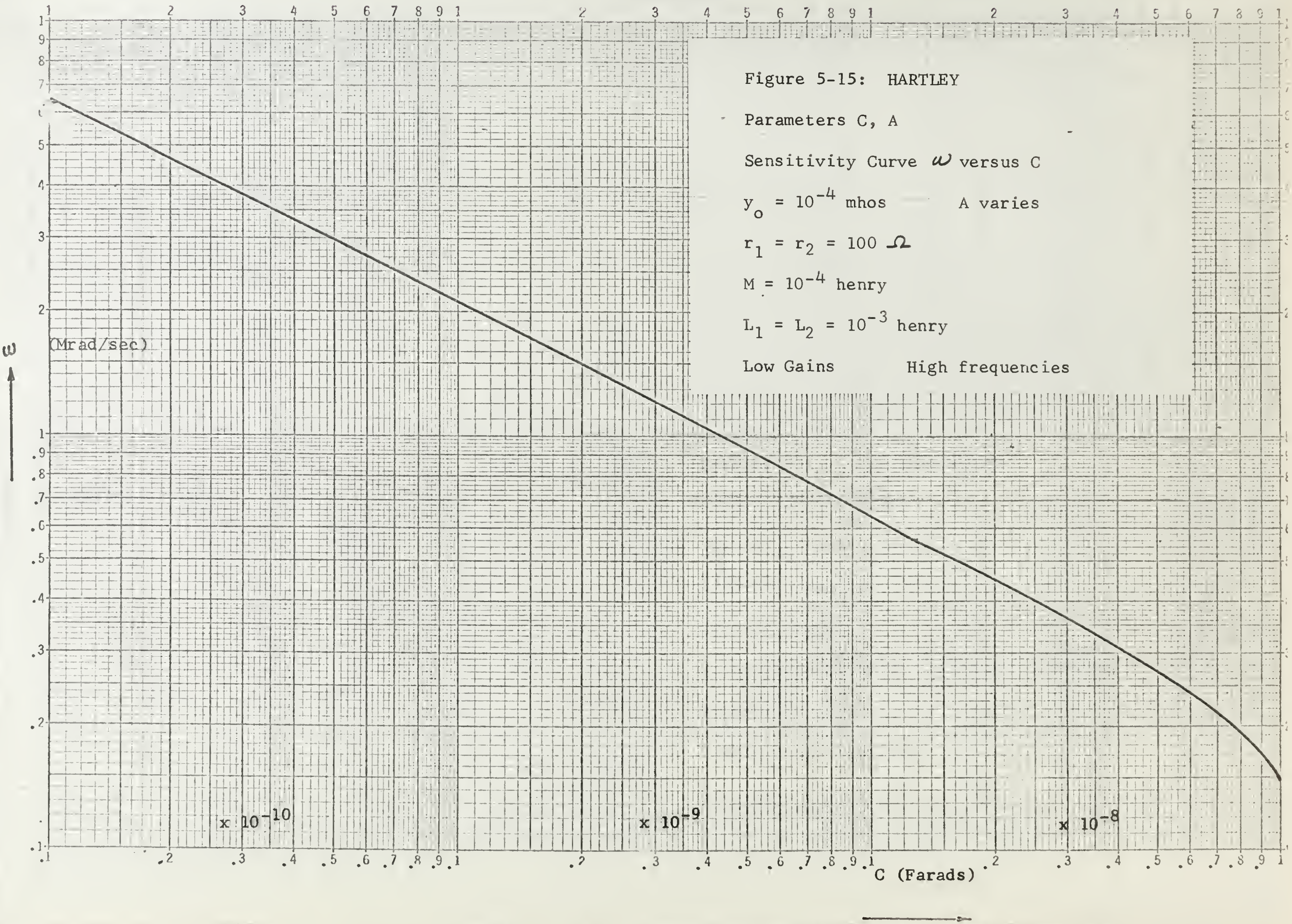


Figure 5-15: HARTLEY
Parameters C, A
Sensitivity Curve ω versus C
$y_o = 10^{-4}$ mhos A varies
$r_1 = r_2 = 100\ \Omega$
$M = 10^{-4}$ henry
$L_1 = L_2 = 10^{-3}$ henry
Low Gains High frequencies
ω (Mrad/sec)
C (Farads)
$\times 10^{-10}$
$\times 10^{-9}$
$\times 10^{-8}$

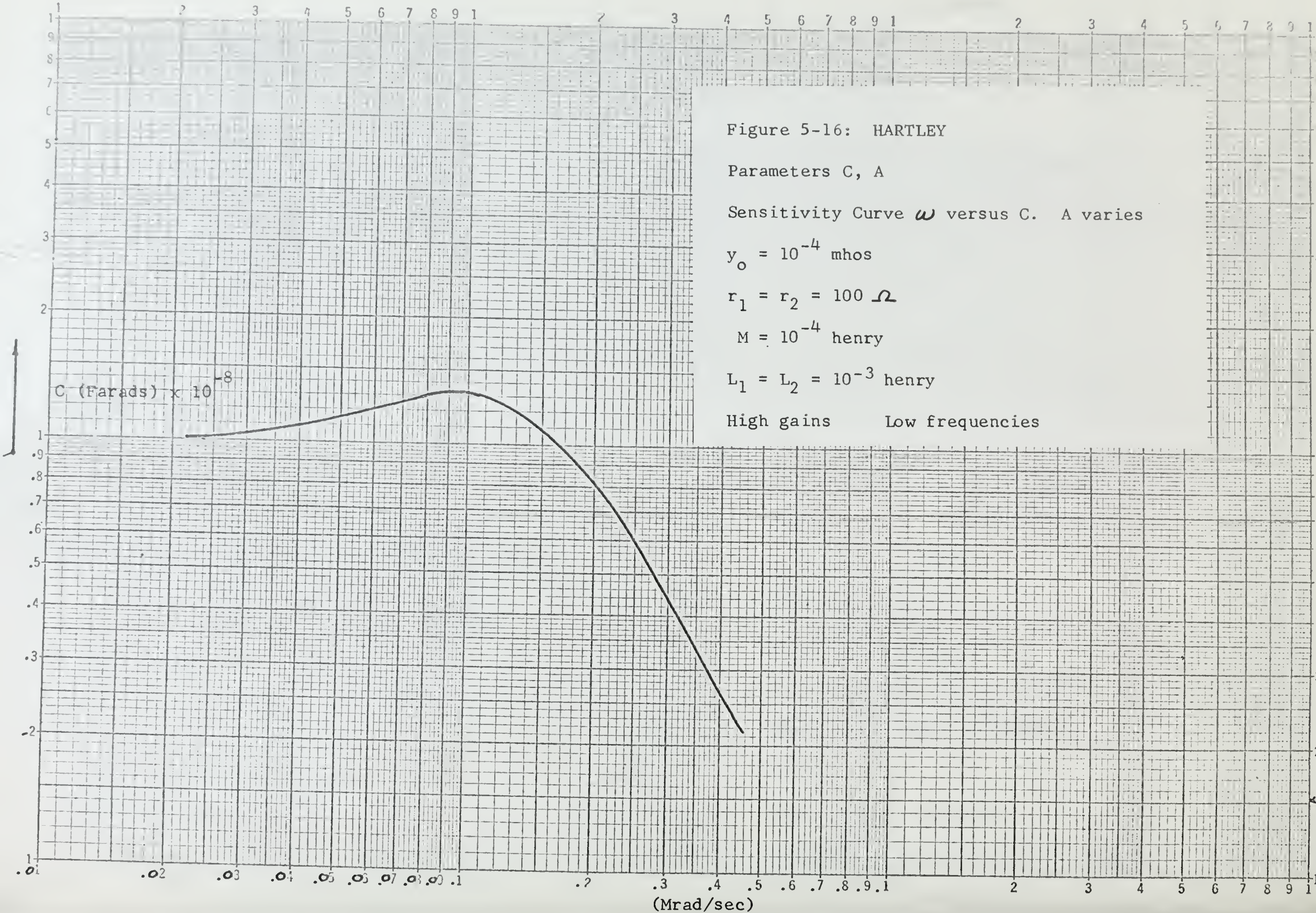


Figure 5-16: HARTLEY

Parameters C, A

Sensitivity Curve ω versus C. A varies

$y_o = 10^{-4}$ mhos

$r_1 = r_2 = 100\ \Omega$

$M = 10^{-4}$ henry

$L_1 = L_2 = 10^{-3}$ henry

High gains Low frequencies

6. Hartley "Dead Zone".

6-1. Analog computer simulation.

In order to study the Hartley oscillator at values of parameters C and L_2 in the "dead zone", we simulate the problem in the analog computer.

An oscillator can be regarded as a feedback problem of gain A and feedback B as shown below:

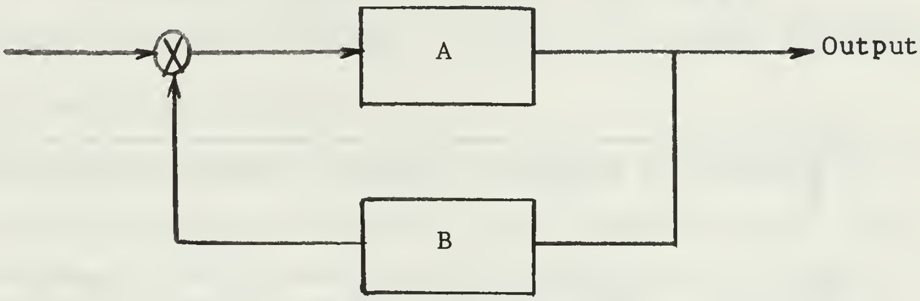


Figure 6-1. Feedback Circuit

The transfer function of this system is:

$$W(S) = \frac{A}{1 - AB}$$

The characteristic equation 1 - AB is already found so we have the transfer function:

$$W(S) = \frac{A}{g_m\left[\frac{1}{Z}(r_1 r_2 - \frac{M}{C} + S^2L_1L_2 - S^2M^2 + SL_1r_2 + SL_2r_1)\right] \cdots}$$

$$\cdots + y_o\left[\frac{1}{Z}(r_1r_2 + \frac{L_2}{C} + S^2L_1L_2 - S^2M^2 + SL_1r_2 + SL_2r_1 + \frac{r_2}{SC})\right] + 1$$

Divide by y_o and multiply by Z and we get:

$$W(S) = \frac{\frac{AZ}{y_o}}{A\left[r_1r_2 - \frac{M}{C} + S^2L_1L_2 - S^2M^2 + SL_1r_2 + SL_2r_1\right] \cdots\cdots}$$

$$\cdots\cdots \overline{+\left[r_1r_2 + \frac{L_2}{C} + S^2L_1L_2 - S^2M^2 + SL_1r_2 + SL_2r_1 + \frac{r_2}{SC}\right] + \frac{Z}{y_o}}$$

By appropriate manipulation of this equation, we get:

$$W(S) = \frac{A\left[S^2C(L_1 + L_2 + 2M) + SC(r_1 + r_2) + 1\right]}{S^3\left[(ACy_oL_1L_2 - ACy_oM^2 + Cy_oL_1L_2 - Cy_oM^2) + \right.} \cdots\cdots$$

$$\cdots\cdots \overline{S^2\left[ACy_oL_1r_2 + ACy_oL_2r_1 + Cy_oL_1r_2 + Cy_oL_2r_1 + C(L_1+L_2+2m)\right] +}$$

$$\cdots\cdots \overline{S\left[ACy_or_1r_2 - Ay_oM + Cy_or_1r_2 + y_oL_2 + C(r_1 + r_2)\right] + r_2y_o + 1}$$

We give the following set of values to the parameters of the circuit and tube.

$y_o = 10^{-4}$ mhos

$r_1 = r_2 = 100\ \Omega$

$M = 10^{-4}$ Henry

$L_1 = 10^{-3}$ Henry

Also, we select a point in the "dead zone" defined by:

$A = 50$

$L_2 = 0.3 \times 10^{-1}$ h

$C = 7 \times 10^{-9}$ f

and the transfer function becomes:

$$W(S) = \frac{50\left[S^2(1.48 \times 10^{-10}) + S(1.4 \times 10^{-6}) + 1\right]}{S^3(0.715 \times 10^{-15}) + S^2(0.224 \times 10^{-9}) + S(3.257 \times 10^{-6}) + 1.01}$$

$$= \frac{S^2(0.74 \times 10^{-8})+S(0.7 \times 10^{-4})+ 0.5 \times 10^2}{S^3(0.715 \times 10^{-15})+S^2(0.224 \times 10^{-9})+S(0.3257 \times 10^{-5}) + 1.01}$$

$$= \frac{0.74 \times 10^{-8} \frac{1}{S} + 0.7 \times 10^{-4} \frac{1}{S^2} + 0.5 \times 10^2 \frac{1}{S^3}}{0.715 \times 10^{-15} + 0.224 \times 10^{-9} \frac{1}{S} + 0.3257 \times 10^{-5} \frac{1}{S^2} + 1.01 \frac{1}{S^3}}$$

$$= \frac{1.035 \times 10^7 \frac{1}{S} + 0.98 \times 10^{11} \frac{1}{S^2} + 0.7 \times 10^{17} \frac{1}{S^3}}{1 - \left[- 0.313 \times 10^6 \frac{1}{S} - 0.455 \times 10^{10} \frac{1}{S^2} - 1.415 \times 10^{15} \frac{1}{S^3}\right]}$$

In order to see oscillations, if any, in the brush recorder we must time scale the computer.

The frequency of oscillations is around $f = 10^4$ cps so if we time scale by 10^4 we represent one cycle in one second (computer time).

$$W(S) = \frac{0.1035 \times 10^4 \frac{1}{S} + 0.98 \times 10^3 \frac{1}{S^2} + 0.7 \times 10^5 \frac{1}{S^3}}{1 - \left[- 0.313 \times 10^2 \frac{1}{S} - 0.455 \times 10^2 \frac{1}{S^2} - 0.1415 \times 10^4 \frac{1}{S^3}\right]}$$

$$\equiv \frac{\frac{a}{S} + \frac{b}{S^2} + \frac{C}{S^3}}{1 - \left[- \frac{d}{S} - \frac{e}{S^2} - \frac{f}{S^3}\right]}$$

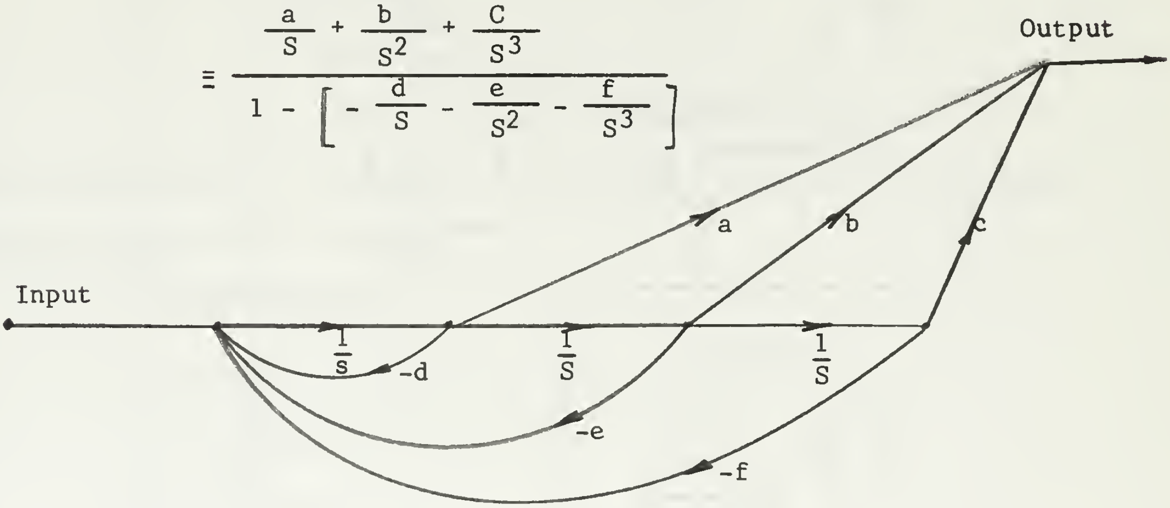
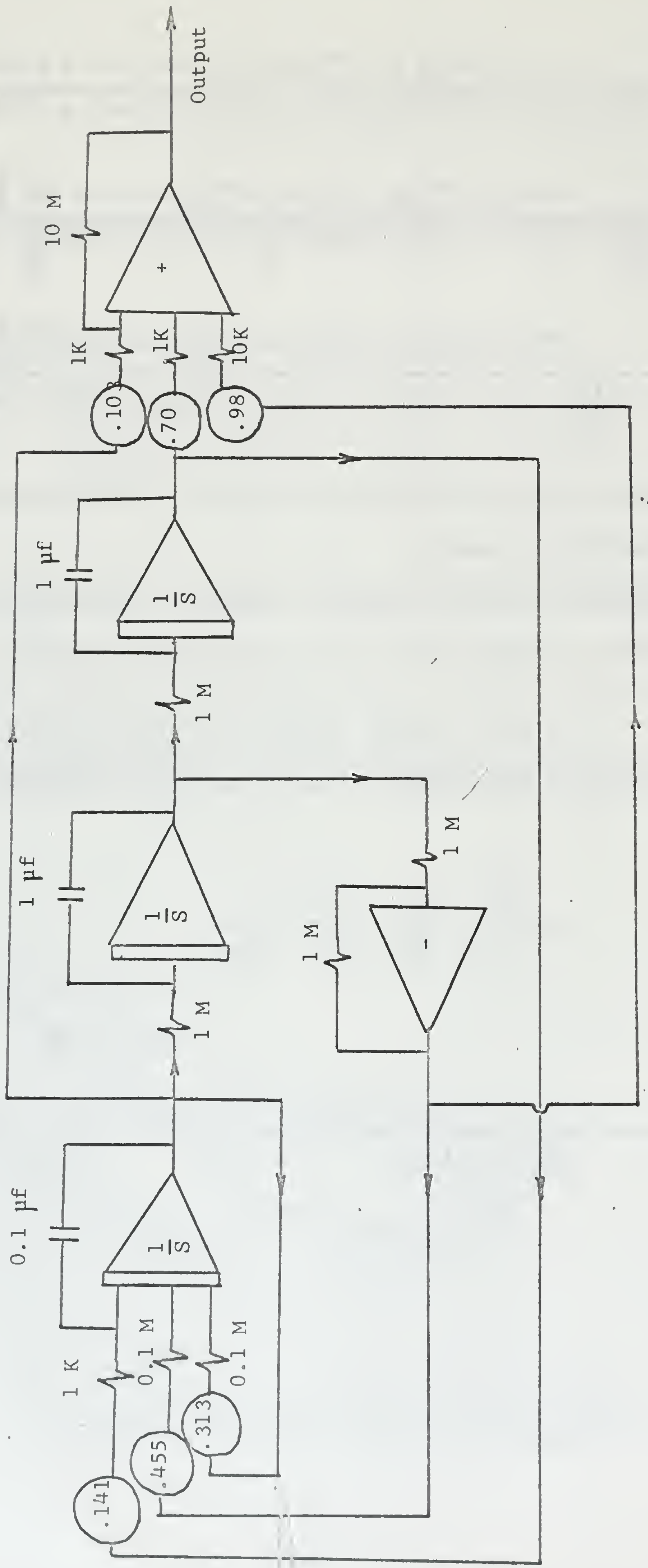


Figure 6-2. Analog Computer Flow Graph

Figure 6-3: COMPUTER DIAGRAM "DEAD ZONE"



Since the gain of the summer is very high we amplitude scale the problem by 10^3. The computer diagram before amplitude scaling appears in Figure 6-3. When we amplitude scale, the input resistors to the summer are to be multiplied by 10^3.

We select a point on the ζ = curve defined by $C = 0.83 \times 10^{-8}$ fd, $L_2 = 0.159 \times 10^{-1}$ h, which corresponds to frequency $\omega = 6.8593 \times 10^4$.

$$S^3\left[(0.83 \times 10^{-8})(.159 \times 10^{-1})(0.51 \times 10^{-1}) - 0.83 \times 10^{-8} \cdot (0.51 \times 10^{-6})\right] + S^2\left[(0.83 \times 10^{-8})(.159 \times 10^{-1})(0.151 \times 10^5) + (.83 \times 10^{-8})(17.1)\right] + S\left[(0.83 \times 10^{-8})(2.51 \times 10^6) + 0.159 \times 10^{-1} - 0.5 \times 10^{-2}\right] + 1.01 \times 10^4$$

Set $S = j\omega$ and equalizing the real part to zero,

$$-\omega^2(0.200 \times 10^{-5} + 0.0142 \times 10^{-5}) = -1.01 \times 10^4$$

$$\omega^2 = \frac{1.01 \times 10^4}{0.2142 \times 10^{-5}} = 4.73 \times 10^9$$

$\omega = 6.86 \times 10^4$ close to the expected value.

For simulationagain, we give the following set of values to the parameters of the circuit and tube.

$y_o = 10^{-4}$ mhos

$r_1 = r_2 = 100\ \Omega$

$M = 10^{-4}$ henry

$L_1 = 10^{-3}$ henry

We select a point on the $\zeta = 0$ curve defined by:

$A = 50$ $C = 0.83 \times 10^{-8}$ f

$L_2 = 0.159 \times 10^{-1}$ h

$$W(S) = \frac{50\left[S^2(0.83 \times 10^{-8})(0.171 \times 10^{-1}) + S(0.83 \times 10^{-8})(2 \times 10^2) + 1\right]}{S^3\left[0.672 \times 10^{-15}\right] + S^2\left[0.214 \times 10^{-9}\right] + S\left[0.316 \times 10^{-5}\right] + 1.01}$$

$$= \frac{S^2(0.71 \times 10^{-8}) + S(0.83 \times 10^{-4} + 0.5 \times 10^2}{S^3\left[0.672 \times 10^{-15}\right] + S^2\left[0.214 \times 10^{-9}\right] + S\left[0.316 \times 10^{-5}\right] + 1.01}$$

$$= \frac{1.06 \times 10^7 \frac{1}{S} + 1.235 \times 10^{11} \frac{1}{S^2} + 0.745 \times 10^{17} \frac{1}{S^3}}{1 - \left[-0.318 \times 10^6 \frac{1}{S} - 0.47 \times 10^{10} \frac{1}{S^2} - 1.5 \times 10^{15} \frac{1}{S^3}\right]}$$

$$= \frac{0.106 \times 10^8 \frac{1}{S} + 0.1235 \times 10^{12} \frac{1}{S^2} + 0.745 \times 10^{17} \frac{1}{S^3}}{1 - \left[-0.318 \times 10^6 \frac{1}{S} - 0.47 \times 10^{10} \frac{1}{S^2} - 0.15 \times 10^{16} \frac{1}{S^3}\right]}$$

Time scale by 10^4

$$W(S) = \frac{0.106 \times 10^4 \frac{1}{S} + 0.1235 \times 10^4 \frac{1}{S^2} + 0.742 \times 10^5 \frac{1}{S^3}}{1 - \left[-0.318 \times 10^2 \frac{1}{S} - 0.47 \times 10^2 \frac{1}{S^2} - 0.15 \times 10^4 \frac{1}{S^3}\right]}$$

By Routh's criterium using the coefficients of the characteristic equation:

$$S^3\left[0.67 \times 10^{-15}\right] + S^2\left[0.212 \times 10^{-9}\right] + S\left[0.316 \times 10^{-5}\right]$$

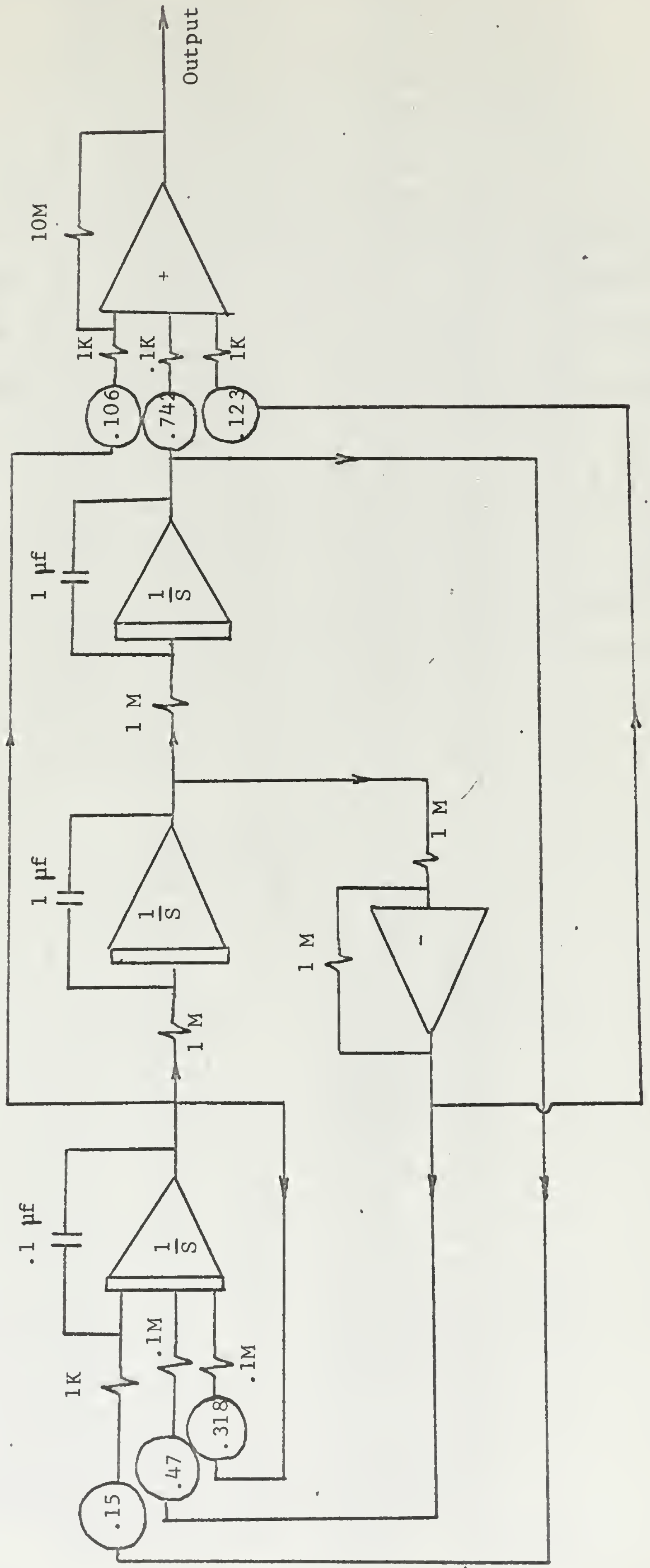
$$+ 1.01 = 0$$

S^3:	0.672×10^{-15}	0.316×10^{-5}
S^2:	0.214×10^{-9}	1.01
S^1:	$\dfrac{(0.214 \times 10^{-9})(0.316 \times 10^{-5}) - (0.672 \times 10^{-18})(1.01)}{0.212 \times 10^{-9}}$	0.0
S^o:	1.01	0.0

In order to have oscillations the S^1 term must go to zero:

$0.675 \times 10^{-15} - 0.68 \times 10^{-15} \doteq 0$. Roots are to the right half of the plane but very close to $j\omega$ axis, and frequency

Figure 6-4: COMPUTER DIAGRAM - OSCILLATORY AREA



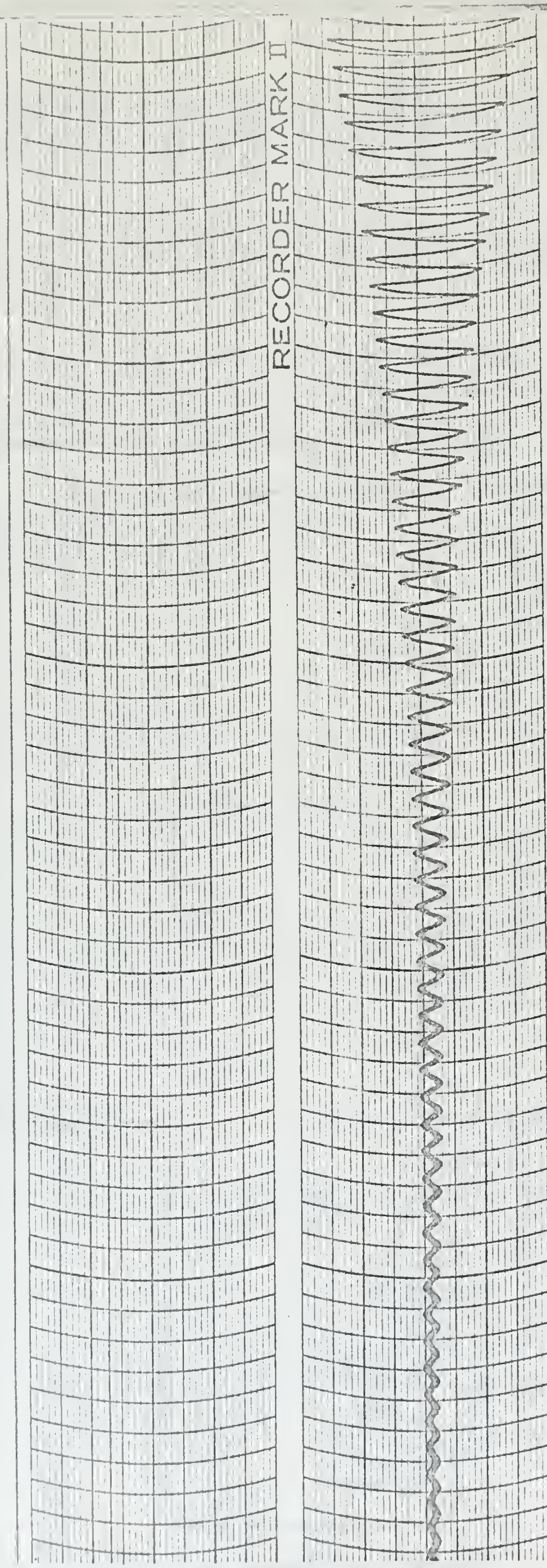


Figure 6-5

ANALOG COMPUTER OUTPUT

OF "DEAD ZONE"

Operation Point:

$A = 50$

$L_2 = 0.2 \times 10^{-1}$ h

$C = 7 \times 10^{-9}$ f

Speed 5 mm/sec

Time scaled by: 10^4

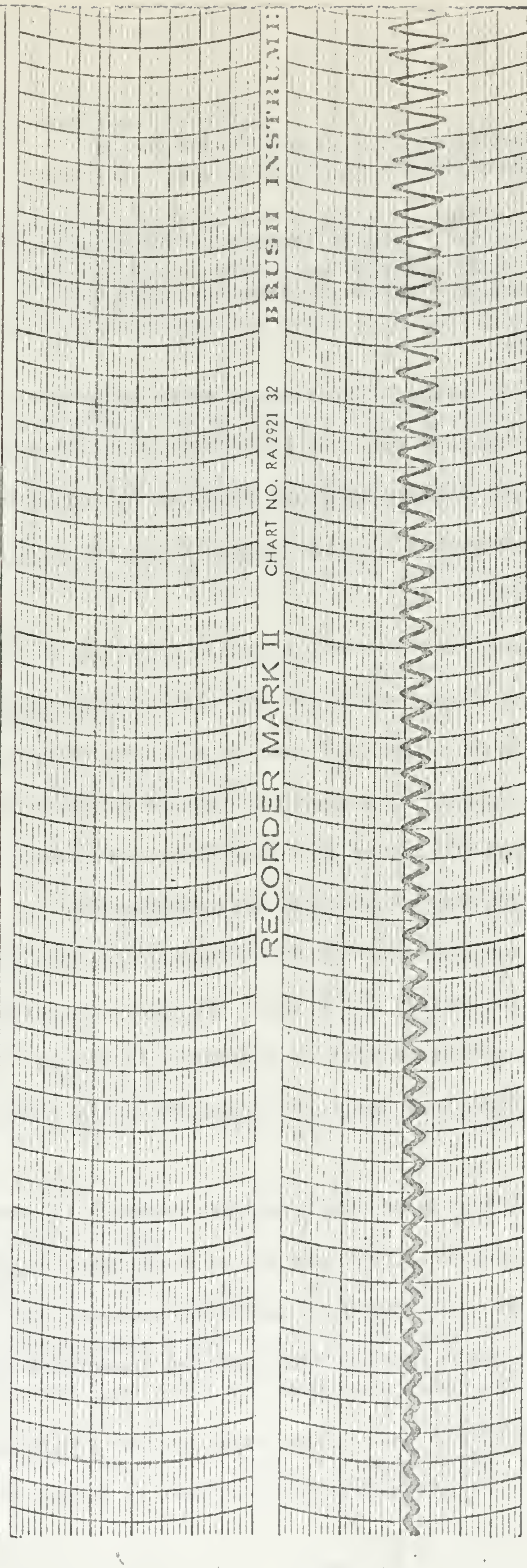


Figure 6-6

ANALOG COMPUTER OUTPUT

OF OSCILLATORY AREA

Operating Point:

$A = 50$

$L_2 = 0.159 \times 10^{-1}$ h

$C = 0.83 \times 10^{-8}$ f

Speed 5 mm/sec

Time scaled by: 10^4

$$\omega^2 = \frac{1.01}{.214 \times 10^{-9}} = 4.68 \times 10^9 = 46.8 \times 10^8$$

$\omega = 6.85 \times 10^4$ almost exactly as shown on the curve in Figure 5-3. The computer diagram is shown in Figure 6-4.

Results of Simulation

The output of the analog simulation is shown on graphs 6-5 and 6-6.

From Figure 6-5 one can see that the system oscillates at about the frequency of 6.87×10^4 rad/sec which is close to the order of frequencies which are to be expected in this region if the curve $\zeta = 0$ for $A = 50$ were continuous.

This point of operation ($A = 50$, $L_2 = 0.2 \times 10^{-1}$ h, $C = 7 \times 10^{-9}$ f) lies to the right half plane, since the oscillations are building up in amplitude. The reason that the amplitude is building up cannot be attributed to the fact that in this area the digital computer gives complex roots for L_2 and C but rather to unavoidable inaccuracy in selecting this point to be on the A = 50 curve by extrapolation.

From Figure 6-6 one can see that the amplitude of oscillations is more stable although ultimately build up as they have to, since it has been found by Routh's criterium that we operate slightly to the right of the $j\omega$ axis. This happens due to plotting and slide rule approximations.

Finally, the building up of amplitude and probably inaccuracy in frequency of oscillations is attributed not only to plotting and slide rule approximations, but also computer and brush recorder calibration and the inaccuracy of the values of resistors and capacitors of the simulation circuit.

6-2. Digital Computer Simulation.

To avoid noise and scaling problems which are inevitable in analog computers, one could simulate the problem for the selected point in the "dead zone" using digital computer by ROUTINE INTEGI.

The characteristic equation taken from analog computer simulation is:

$$S^3(0.715 \times 10^{-15}) + S^2(0.224 \times 10^{-9}) + S(3.257 \times 10^{-6}) + 1.01 = 0$$

$$S^3 + (0.313 \times 10^6)\, S^2 + (0.455 \times 10^{10})S + 0.1415 \times 10^{16} = 0$$

$$\dddot{X} + (0.313 \times 10^6)\, \ddot{X} + (0.455 \times 10^{10})\dot{X} + 0.1415 \times 10^{16} X = 1$$

Initial condition

$$\text{Let } X(1) \triangleq X$$

$$X(2) = \dot{X}(1) = \dot{X}$$

$$X(3) = \dot{X}(2) = \ddot{X}$$

$$\dot{X}(3) \triangleq \dddot{X} = -C(1)\, X(1) - C(2)\, X(2) - C(3)\, X(3) + C(4).$$

Equation to be solved:

$$\dot{X}(1) = X(2)$$

$$\dot{X}(2) = X(3)$$

$$\dot{X}(3) = -C(1)*X(1) - C(2)*X(2) - C(3)*X(3) + C(4)$$

Since $\omega \doteq 6.86 \times 10^4$, $f = 1.09 \times 10^4$, $T \doteq 0.92 \times 10^{-4}$ sec, we select the time interval $\sim \frac{T}{100} = 0.9 \times 10^{-6}$.

Initial time = 0 Final time = 5×10^{-4} sec.

From the digital computer output of about five cycles (Figure 6-7) we see that the circuit in the "dead zone" oscillates. Again, the fact that oscillations seem to converge is immaterial, since we are interested only in seeing if the circuit oscillates.

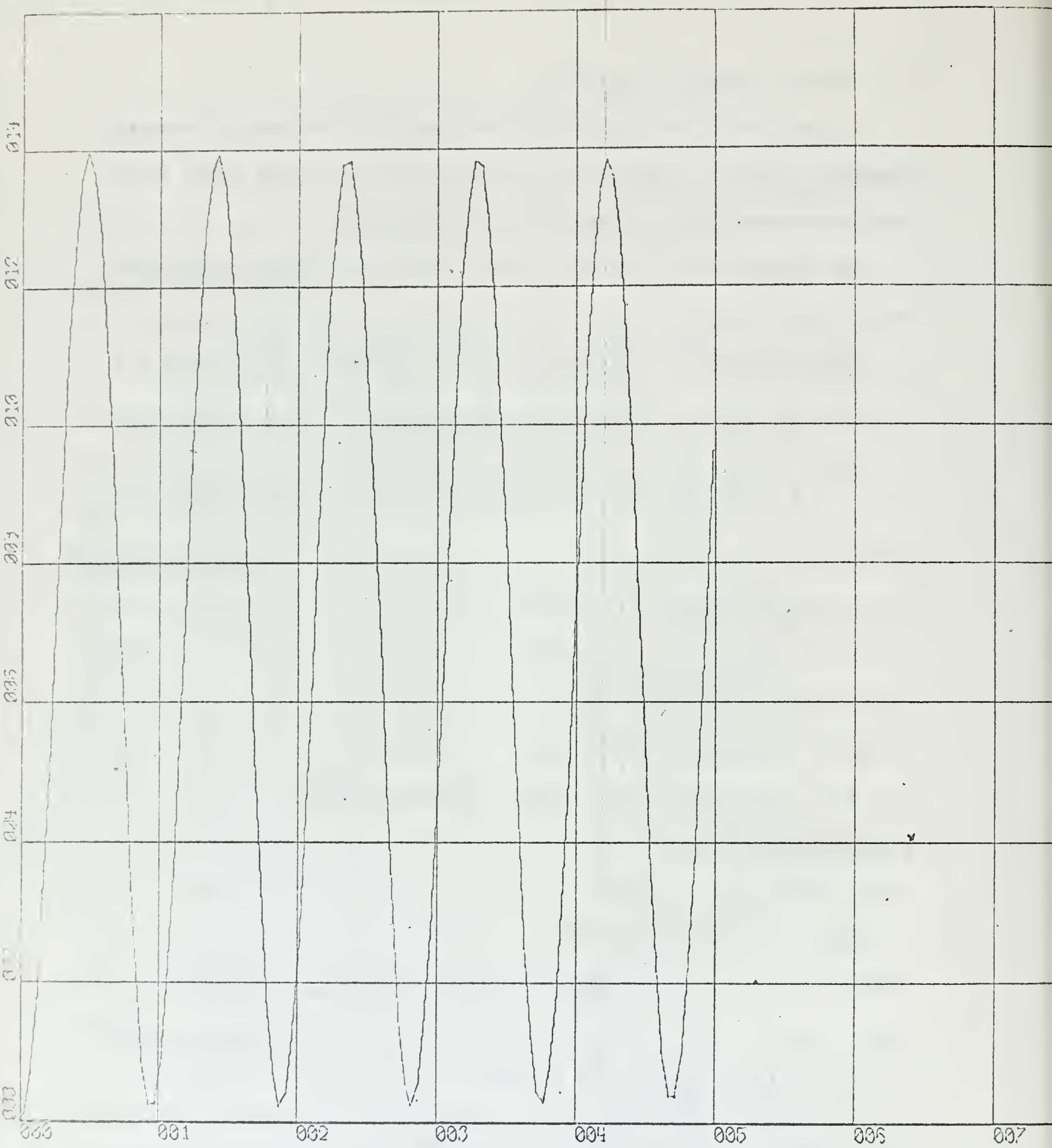


Figure 6-7: DIGITAL COMPUTER OUTPUT
OF "DEAD ZONE"

X-SCALE = 1.00E-04 UNITS/INCH.
Y-SCALE = 2.00E-16 UNITS/INCH.

PROGRAM DEAD ZONE ZORBAS

RUN 1 THETA VS TIME

6-3. Routh's criterion test of points.

Working backwards we leave gain A as variable in the case of the point in the "dead zone" and solve for A at the limit of stability (on the $j\omega$ axis).

Characteristic equation:

$$S^3\left[A(L_1L_2 - M^2)Cy_o + (L_1L_2 - M^2)Cy_o\right] +$$

$$S^2\left[A(L_1r_2 + L_2r_1)Cy_o + C(y_oL_1r_2 + y_oL_2r_1 + L_1 + L_2 + 2M)\right] +$$

$$S\left[A(Cr_1r_2 - M)y_o + Cy_or_1r_2 + y_oL_2 + C(r_1 + r_2)\right] + (r_2y_o + 1) = 0$$

Putting values:

$y_o = 10^{-4}$ mhos

$r_1 = r_2 = 100\ \Omega$

$M = 10^{-4}$ henry

$L_1 = 10^{-3}$

$L_2 = 0.2 \times 10^{-1}$ h

$C = 0.7 \times 10^{-8}$ f

we get

$$S^3\left[A(0.2 \times 10^{-4} - 10^{-8})(.7 \times 10^{-8})(10^{-4}) + (0.2 \times 10^{-4} - 10^{-8})\cdot\right.$$

$$\left.\cdot(.7 \times 10^{-8})(10^{-4})\right] + S^2\left[A(2.1)(.7 \times 10^{-8})(10^{-4}) + (.7 \times 10^{-8})\cdot\right.$$

$$\left.\cdot(10^{-5} + 0.2 \times 10^{-3} + 10^{-3} + .2 \times 10^{-1} + .2 \times 10^{-3})\right] + S\left[A(.7 \times 10^{-4}\right.$$

$$\left.- 10^{-4})(10^{-4}) + (.7 \times 10^{-8}) + .2 \times 10^{-5} + (0.7 \times 10^{-8})(2 \times 10^2)\right]$$

$$+ (10^{-2} + 1) = 0$$

$$S^3\left[(0.14 \times 10^{-16})A + (0.14 \times 10^{-16})\right] +$$

$$S^2\left[(0.147 \times 10^{-11})A + (0.7 \times 10^{-8})(0.2141 \times 10^{-1})\right]$$

$$+ S\left[(-0.3 \times 10^{-8})A + 0.34 \times 10^{-5}\right] + 1.01$$

Routh's criterium:

$$S^3 \;:\; \left[(0.14 \times 10^{-16})A + 0.14 \times 10^{-16}\right] \qquad \left[(-0.3 \times 10^{-8})A + 0.34 \times 10^{-5})\right]$$

$$S^2 \;:\; \left[(0.14 \times 10^{-11})A + 0.15 \times 10^{-9}\right] \qquad 1.01$$

For the limit of stability

$$\left[(-0.3 \times 10^{-8})A + 0.34 \times 10^{-5}\right]\cdot\left[(0.14 \times 10^{-11})A + 0.15 \times 10^{-9}\right] -$$

$$- \left[(0.14 \times 10^{-16})A + 0.14 \times 10^{-16}\right]\cdot\left[1.01\right] = 0$$

$$-A^2(0.42 \times 10^{-20}) + A(-0.45 \times 10^{-18} + 0.476 \times 10^{-17}) + 0.51 \times 10^{-15}$$

$$-A(0.141 \times 10^{-16}) - 0.141 \times 10^{-16} = 0$$

$$A^2(0.42 \times 10^{-20}) + A(9.79 \times 10^{-18}) - 4.97 \times 10^{-16} = 0$$

$$A^2(0.42 \times 10^{-4}) + A(9.97 \times 10^{-2}) - 4.97 = 0$$

$$A^2 + 2.33 \times 10^3 A - 1.18 \times 10^5 = 0$$

$$A = \frac{-2.33 \times 10^3 \pm \left[5.4 \times 10^6 + 4.72 \times 10^5\right]^{1/2}}{2} =$$

$$\frac{-2.33 \times 10^3 \pm 2.42 \times 10^3}{2} = \frac{.09 \times 10^3}{2} \qquad \frac{90}{2} = \underline{\underline{45}}$$

This certifies the fact that the point lies to the right half plane, because in order to come on the $j\omega$ axis we must reduce gain from 50 to 45. Plug A back into the S^2 row and solve for frequency and we get:

$$W^2 = \frac{1.01}{(0.14 \times 10^{-11})45 + 0.15 \times 10^{-9}} = \frac{1.01}{0.213 \times 10^{-9}} = 4.7 \times 10^9$$

$$W = 6.85 \times 10^4 \text{ rad/sec}$$

Now take a point to the right of the previous one and try to get

$A = 50.$

The point is defined:

$$L_2 = .205 \times 10^{-1}\ h$$

$$C = .7 \times 10^{-8}\ f$$

Characteristic equation becomes:

$$S^3\Big[A(0.205 \times 10^{-4} - 10^{-8})(.7 \times 10^{-8})(10^{-4}) + (0.205 \times 10^{-4}$$

$$-10^{-8})(.7 \times 10^{-8})(10^{-4})\Big] + S^2\Big[A(2.15)(.7 \times 10^{-8})(10^{-4}) +$$

$$(.7 \times 10^{-8})(10^{-5} + 0.205 \times 10^{3} + .205 \times 10^{-1} + 10^{-3} +$$

$$.2 \times 10^{-3})\Big] + S\Big[A(.7 \times 10^{-4} - 10^{-4})(10^{-4}) + (.7 \times 10^{-8}) +$$

$$.205 \times 10^{-5} + (.7 \times 10^{-8})(2 \times 10^{2})\Big] + (10^{-2} + 1) = 0$$

$$S^3\Big[A(0.144 \times 10^{-16}) + (0.144 \times 10^{-16})\Big] + S^2\Big[A(0.151 \times 10^{-11})$$

$$+ (.7 \times 10^{-8})(.2195 \times 10^{-1})\Big] + S\Big[A(-0.3 \times 10^{-8}) + 0.345 \times 10^{-5}\Big]$$

$$+ 1.01$$

S^3	:	$A(0.144 \times 10^{-16}) + (0.144 \times 10^{-16})$	$A(-0.3 \times 10^{-8}) + .345 \times 10^{-5}$
S^2	:	$A(.151 \times 10^{-11}) + (0.154 \times 10^{-9})$	1.01

For the limit of stability:

$$\Big[A(-.3 \times 10^{-8}) + (.345 \times 10^{-5})\Big]\cdot\Big[A(.151 \times 10^{-11} + .154 \times 10^{-9}$$

$$-\Big[A(.144 \times 10^{-16}) + (.144 \times 10^{-16})\Big]\cdot\Big[1.01\Big] = 0$$

$$A^2\Big[.454 \times 10^{-20}\Big] + A\Big[.975 \times 10^{-17}\Big] - .547 \times 10^{-15} = 0$$

$$A^2 + 2.15 \times 10^{3}A - 1.21 \times 10^{5} = 0$$

$$A = \frac{-2.15 \times 10^3 \pm \left[4.62 \times 10^6 + 4.84 \times 10^5\right]^{1/2}}{2}$$

$$= \frac{-2.15 \pm 2.26}{2} \times 10^3$$

$$A = \frac{110}{2} = 55$$

$$\omega^2 = \frac{1.01}{(55)(.151 \times 10^{-11}) + (.154 \times 10^{-9})} = \frac{1.01}{.237} \times 10^9$$

$$= 4.26 \times 10^9$$

$$\omega = 6.525 \times 10^4 \text{ rad/sec.}$$

Interpolating between values of L_2 we define a point:

$$\text{Point A} \begin{cases} L_2 = .203 \times 10^{-1} \text{ h} \\ C = .7 \times 10^{-8} \text{ f} \end{cases}$$

$$s^3\left[A(0.203 \times 10^{-4})(.7 \times 10^{-8})(10^{-4}) + (.203 \times 10^{-4})(.7 \times 10^{-8})(10^{-4})\right] +$$

$$s^2\left[A(2.13)(.7 \times 10^{-8})(10^{-4}) + (.7 \times 10^{-8})(10^{-5} + 0.203 \times 10^{-3} + .203 \times 10^{-1} +\right.$$

$$\left.10^{-3} + .203 \times 10^{-3})\right] + s\left[A(.7 \times 10^{-4})(10^{-4}) + (.7 \times 10^{-8}) + .203 \times 10^{-5}\right.$$

$$\left.+ (.7 \times 10^{-8})(2 \times 10^2)\right] + (10^{-2} + 1) = 0$$

$$s^3\left[A(.142 \times 10^{-16}) + (.142 \times 10^{-16})\right] +$$

$$s^2\left[A(.149 \times 10^{-11}) + (.7 \times 10^{-8})(.217 \times 10^{-1})\right]$$

$$s\left[A(-.3 \times 10^{-8}) + .343 \times 10^{-5}\right] + 1.01 = 0$$

Routh array:

s^3 : $A(.142 \times 10^{-16}) + (.142 \times 10^{-16})$ $\quad$ $A(-.3 \times 10^{-8}) + .343\times10^{-5}$

s^2 : $A(.149 \times 10^{-11}) + .152 \times 10^{-9}$ $\quad$ 1.01

At the limit of stability:

$$\left[A(.149 \times 10^{-11}) + .152 \times 10^{-9}\right]\cdot\left[A(-.3 \times 10^{-8}) + .343 \times 10^{-5}\right]$$

$$-\left[A(.142 \times 10^{-16}) + (.143 \times 10^{-16})\right]\cdot\left[1.01\right] = 0$$

$$A^2\left[.447 \times 10^{-20}\right] + A\left[.9736 \times 10^{-17}\right] - .507 \times 10^{-15} = 0$$

$$A^2 + A(2.18 \times 10^3) - 1.17 \times 10^5 = 0$$

$$A = \frac{2.18 \pm \left[4.75 \times 10^6 + 4.68 \times 10^5\right]^{1/2}}{2} \quad \frac{-2.18 \pm 2.28}{2} \times 10^3$$

$$A = \frac{100}{2} = 50$$

So this point falls on the $\zeta = 0$ curve for A = 50. The corresponding frequency is:

$$\omega^2 = \frac{1.01}{(50)(.149 \times 10^{-11}) + .152 \times 10^{-9}} = \frac{1.01 \times 10^9}{.226}$$

$$\omega = 6.68 \times 10^4 \text{ rad/sec}$$

We take another point defined by:

$$L_2 = .22 \times 10^{-1} \text{ h}$$

$$C = .64 \times 10^{-8} \text{ f}$$

The characteristic equation becomes:

$$S^3\left[A(.22 \times 10^{-4} - 10^{-8})(.64 \times 10^{-8})(10^{-4}) + (.22 \times 10^{-4} - 10^{-8})(.64 \times 10^{-8})\cdot\right.$$

$$\left.\cdot(10^{-4})\right] + S^2\left[A(2.3)(.64 \times 10^{-8})(10^{-4}) + (.64 \times 10^{-8})(10^{-5} + .22 \times 10^{-3}\right.$$

$$\left.+ .22 \times 10^{-1} + 10^{-3} + .2 \times 10^{-3})\right] + S\left[A(.64 \times 10^{-4} - 10^{-4})(10^{-4}) +\right.$$

$$\left.(.64 \times 10^{-8}) + .22 \times 10^{-5} + (.64 \times 10^{-8})(2 \times 10^2)\right] + (10^{-2} + 1) = 0$$

$$s^3\left[A(.141 \times 10^{-16}) + (.141 \times 10^{-16})\right] + s^2\left[A(.147 \times 10^{-11}) + (.64 \times 10^{-8})\right.$$

$$\left.(.2342 \times 10^{-1})\right] + s\left[A(-.36 \times 10^{-8}) + (.348 \times 10^{-5})\right] + 1.01 = 0$$

Routh array:

$$s^3 : A(.141 \times 10^{-16}) + (.141 \times 10^{-16}) \qquad A(-36\times10^{-8}) + (.348\times10^{-5})$$

$$s^2 : A(.147 \times 10^{-11}) + (.15 \times 10^{-9}) \qquad 1.01$$

For the limit of stability:

$$\left[A(.147 \times 10^{-11}) + (.15 \times 10^{-9})\right]\cdot\left[A(-.36 \times 10^{-8}) + (.348 \times 10^{-5})\right]$$

$$- \left[A(.141 \times 10^{-16}) + (.141 \times 10^{-16})\right]\cdot\left[1.01\right] = 0$$

$$A^2(.529 \times 10^{-20}) + A(.958 \times 10^{-17}) - 5.36 \times 10^{-16} = 0$$

$$A^2 + 1.81 \times 10^3 A - 1.01 \times 10^3 = 0$$

$$A = \frac{-1.81 \times 10^3 \pm \left[3.28 \times 10^6 + 40.4 \times 10^4\right]^{1/2}}{2} \quad \frac{-1.81 \pm 1.93}{2} \times 10^3$$

$$= \frac{110}{2} = 55$$

We take the next approximation, a point defined:

$$\text{point B}\begin{cases} L_2 = .217 \times 10^{-1} \text{ h} \\ C = .64 \times 10^{-8} \text{ f} \end{cases}$$

$$s^3\left[A(.217 \times 10^{-4})(.64 \times 10^{-8})(10^{-4}) + (.217 \times 10^{-4})(.64 \times 10^{-8})(10^{-4})\right]$$

$$+ s^2\left[A(2.27)(.64 \times 10^{-8})(10^{-4}) + (.64 \times 10^{-8})(10^{-5} + .217 \times 10^{-3} + .217\right.$$

$$\left.\times 10^{-1} + 10^{-3} + .2 \times 10^{-3})\right] + s\left[A(.64 \times 10^{-4} - 10^{-4}) + (.64 \times 10^{-8}) +\right.$$

$$\left..217 \times 10^{-5} + (.64 \times 10^{-8})(2 \times 10^{2})\right] + 1.01 = 0$$

$$s^3\left[A(.139 \times 10^{-16}) + (.139 \times 10^{-16})\right] +$$

$$+ S^2 \left[A(.145 \times 10^{-11}) + (.64 \times 10^{-8})(.231 \times 10^{-1})\right] +$$

$$S \left[A(-.36 \times 10^{-8}) + (.345 \times 10^{-5})\right] + 1.01 = 0$$

At the limit of stability:

<u>Routh array</u>:

$$S^3 : \quad A(.139 \times 10^{-16}) + (.139 \times 10^{-16}) \qquad A(-.36 \times 10^{-5}) + (.345 \times 10^{-5})$$

$$S^2 : \quad A(.145 \times 10^{-11}) + (.148 \times 10^{-9}) \qquad 1.01$$

$$\left[A(.145 \times 10^{-11}) + (.148 \times 10^{-9})\right] \cdot \left[A(-.36 \times 10^{-8}) +\right.$$

$$\left.(.345 \times 10^{-5})\right] - \left[A(.139 \times 10^{-16}) + (.139 \times 10^{-16})\right] \cdot$$

$$\cdot \left[1.01\right] = 0$$

$$A^2 \left[.522 \times 10^{-20}\right] + A\left[.958 \times 10^{-17}\right] - .524 \times 10^{-15} = 0$$

$$A^2 + A(1.885 \times 10^3) - 10^5 = 0$$

$$A = \frac{-1.885 \pm \left[3.55 \times 10^6 + 4 \times 10^5\right]^{1/2}}{2} = \frac{-1.885 \pm 1.985}{2} 10^3$$

$$A = \frac{100}{2} = 50$$

So this point also falls on top of $\zeta = 0$ curve for A = 50.

Frequency at this point is

$$\omega^2 = \frac{1.01}{(50)(.147 \times 10^{-11}) + (.15 \times 10^{-9})} + \frac{1.01}{.223} \times 10^9$$

$\omega = 6.74 \times 10^4$ rad/sec, which is higher than $\omega = 6.68 \times 10^4$ rad/sec found for the previous point.

We test for another point on A = 50 curve, defined by:

$$\left.\begin{aligned} L_2 &= .19 \times 10^{-1} \text{ h} \\ C &= .74 \times 10^{-8} \text{ f} \end{aligned}\right\} \text{point C of Figure 6-8}$$

$$S^3\left[A(.19 \times 10^{-4})(.74 \times 10^{-8})(10^{-4}) + (.19 \times 10^{-4})(.74 \times 10^{-8})(10^{-4})\right]$$

$$+ S^2\left[A(2)(.74 \times 10^{-8})(10^{-4}) + (.74 \times 10^{-8})(10^{-5} + .19 \times 10^{-3} + .19 \times 10^{-1} + 10^{-3} + .2 \times 10^{-3})\right] + S\left[A(.74 \times 10^{-4} - 10^{-4})(10^{-4}) + (.74 \times 10^{-8}) + .19 \times 10^{-5} + (.75 \times 10^{-8})(2 \times 10^{2})\right] + 1.01 = 0$$

$$S^3\left[A(.141 \times 10^{-16}) + (.141 \times 10^{-16})\right] +$$

$$S^2\left[A(.148 \times 10^{-11}) + (.74 \times 10^{-8})(.2039 \times 10^{-1})\right]$$

$$S\left[A(-.26 \times 10^{-8}) + (.338 \times 10^{-5})\right] + 1.01 = 0$$

Routh array:

S^3 : $A(.141 \times 10^{-16}) + (.141 \times 10^{-16})$ $\quad A(-.26 \times 10^{-8}) + .338\times10^{-5}$

S^2 : $A(.148 \times 10^{-11}) + (.151 \times 10^{-9})$ $\quad 1.01$

$$\left[A(.148 \times 10^{-11}) + (.151 \times 10^{-9})\right]\cdot\left[A(-.26 \times 10^{-8}) + .338 \times 10^{-5}\right] - \left[A(.141 \times 10^{-16}) + (.141 \times 10^{-16})\right]\cdot\left[1.01\right] = 0$$

$$A^2\left[.385 \times 10^{-20}\right] + A\left[.964 \times 10^{-17}\right] - .594 \times 10^{-15} = 0$$

$$A^2 + A(2.5 \times 10^3) - 1.36 \times 10^5 = 0$$

$$A = \frac{-2.5 \pm \left[6.25 \times 10^6 + 5.44 \times 10^5\right]^{1/2}}{2} = \frac{-2.5 \pm 2.6}{2} \times 10^3$$

$$A = \frac{100}{2} = 50$$

Frequency for this point is:

$$\omega^2 = \frac{1.01}{(50)(.148 \times 10^{-11}) + (.151 \times 10^{-4})} = \frac{1.01}{.225 \times 10^{-9}}$$

$$\omega = 6.7 \times 10^4 \text{ rad/sec}$$

So we can conclude that the curves, although seemingly interrupted due to the complex values of parameters L_2, C, in reality are interconnected. There is a minimum in frequency however, and below this minimum we have no oscillations.

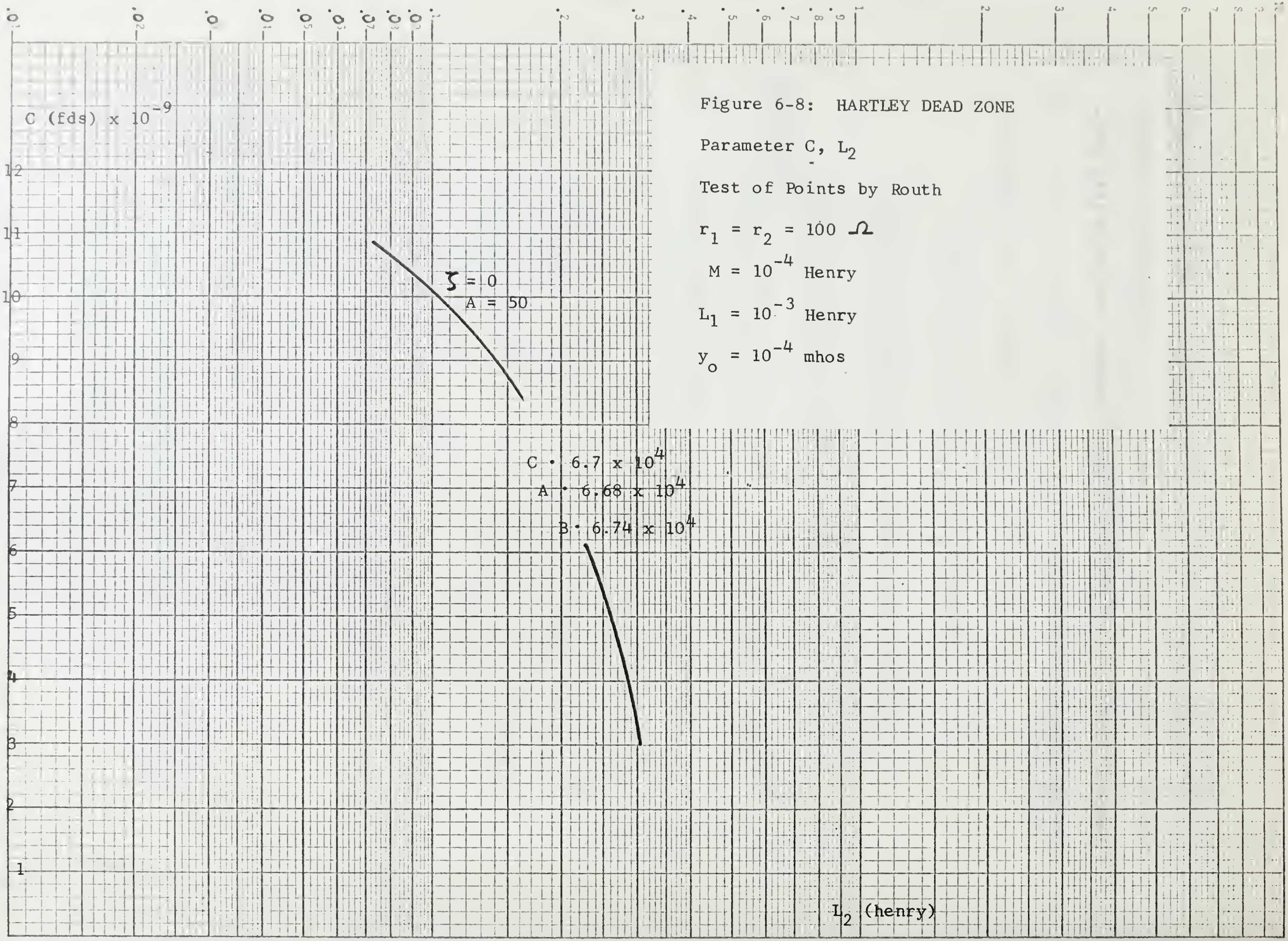


Figure 6-8: HARTLEY DEAD ZONE

Parameter C, L_2

Test of Points by Routh

$r_1 = r_2 = 100\ \Omega$

$M = 10^{-4}$ Henry

$L_1 = 10^{-3}$ Henry

$y_o = 10^{-4}$ mhos

7. Tube Tuned Plate Oscillator

7-1. Derivation of the characteristic equation.

It has been found [2] that:

$$Z_i = \frac{1}{\omega^2 C^2 Z} - j\frac{1}{\omega C} = -\frac{1}{S^2C^2Z} + \frac{1}{SC} \quad \text{letting } S = j\omega$$

$$Z_f = \frac{M}{CZ}$$

$$Z = r + SL + \frac{1}{SC} = \frac{SCr + S^2LC + 1}{SC}$$

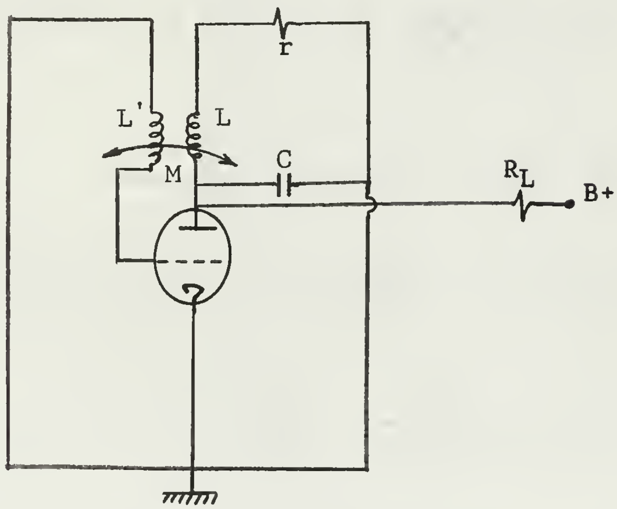


Figure 7-1. Tuned plate oscillator

and equivalently:

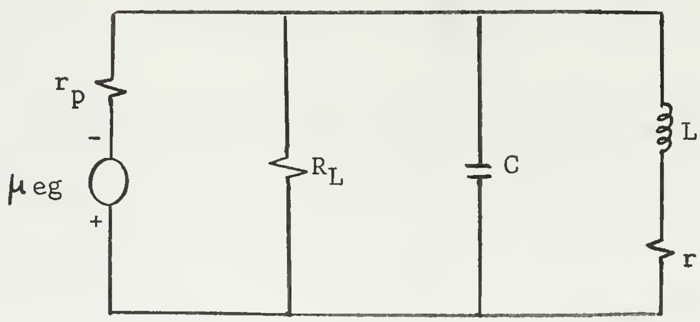


Figure 7-2. Tuned plate equivalent circuit

Putting those values in the relation (18), we get:

$$g_m \frac{M}{CZ} + y_o(- \frac{1}{S^2C^2Z} + \frac{1}{SC}) + 1 = 0$$

$$g_m \frac{M}{CZ} + y_o(\frac{SCZ - 1}{S^2C^2Z}) + 1 = 0$$

$$S^2 g_m MC + y_o CZS - y_o + S^2C^2Z = 0. \quad \text{Putting } Z = \frac{SCR + S^2LC + 1}{SC}$$

$$S^2 g_m MC + \frac{SCr + S^2LC + 1}{SC}\left[y_o CS + S^2C^2\right] - y_o = 0$$

$$S^2CL + S(g_m M + Ly_o + Cr) + (y_o r + 1) = 0$$

where

$$y_o = \frac{1}{Z_o} = \frac{1}{r_p//R_L} = \frac{r_p + R_L}{r_p R_L}$$

Tube voltage gain $A = \dfrac{\mu R_L}{r_p + R_L}$ since $\mu = r_p g_m$

$$A = g_m \frac{r_p R_L}{r_p + R_L} \qquad \frac{g_m}{y_o}$$

Divide by y_o so that we have a relation involving gain which usually comes along with the amplifier.

$$S^2 \frac{CL}{y_o} + S(AM + L + \frac{Cr}{y_o}) + (r + \frac{1}{y_o}) = 0$$

7-2. Parameter plane curves C versus r for different gains.

Let us give some typical values to the different components of the circuit:

$y_o = 10^{-4}$ mhos

$L = 10^{-3}$ h

$M = -10^{-4}$ h

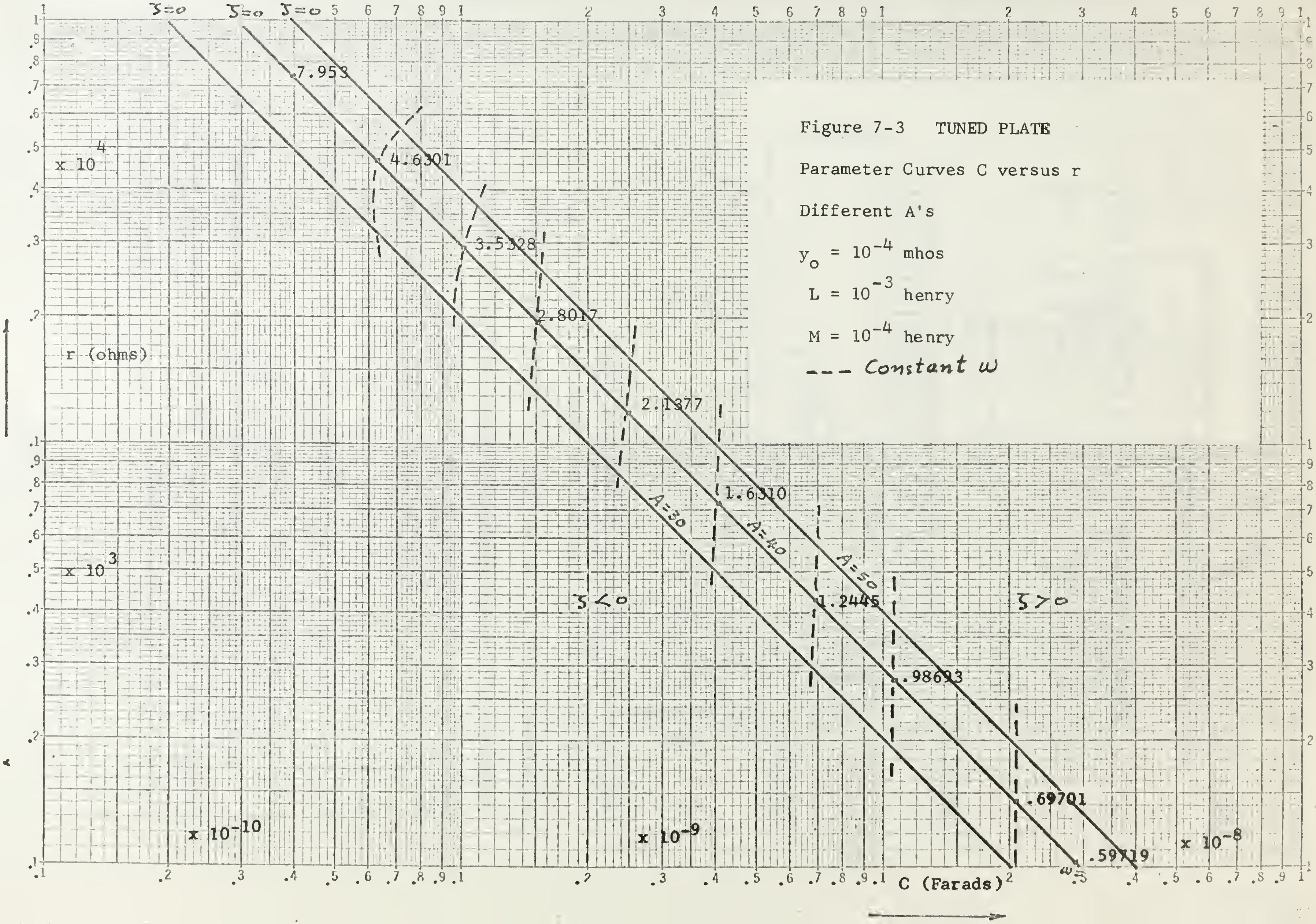


Figure 7-3 TUNED PLATE
Parameter Curves C versus r
Different A's
$y_o = 10^{-4}$ mhos
$L = 10^{-3}$ henry
$M = 10^{-4}$ henry
--- Constant ω
r (ohms)
C (Farads)
x 10^4
x 10^3
x 10^{-10}
x 10^{-9}
x 10^{-8}
ζ=0
ζ=0
ζ=0
ζ<0
ζ>0
A=30
A=40
A=50
7.953
4.6301
3.5328
2.8017
2.1377
1.6310
1.2445
.98693
.69701
.59719

For A = 30

Characteristic equation becomes:

$$S^2(10C) + S(-3 \times 10^{-3} + 10^{-3} + 10^4 Cr) + (r + 10^4) = 0$$

$$S^2(.1 \times 10^2 C) + S\left[.1 \times 10^5 Cr - .2 \times 10^{-2}\right] + \left[.1 \times 10^1 r + .1 \times 10^5\right] = 0$$

For A = 40

$$S^2(.1 \times 10^2 C) + S\left[.1 \times 10^5 Cr - .3 \times 10^{-2}\right] + \left[.1 \times 10^1 r + .1 \times 10^5\right] = 0$$

For A = 50

$$S^2(.1 \times 10^2 C) + S\left[.1 \times 10^5 Cr - .4 \times 10^{-2}\right] + \left[.1 \times 10^1 r + 0.1 \times 10^5\right] = 0$$

Results for Tuned Plate: Parameters C versus r:

The $\zeta = 0$ curves have the same shape for different gains and they are straight lines on log-log paper, so logarithmic variation of one parameter requires logarithmic variation of the other also, in order that the system will operate on the $j\omega$ axis.

The constant ω curves are almost vertical for the lower part of the $\zeta = 0$ curves and they become distorted at higher ω like $\omega >$ 3.5928 x 10 rad/sec as shown in Figure 7-3.

7-3. Transistor Tuned Plate (Collector) Oscillator:

To derive the characteristic equation of this circuit we work differently because the already derived equation (18) does not hold in general for the transistor circuit.

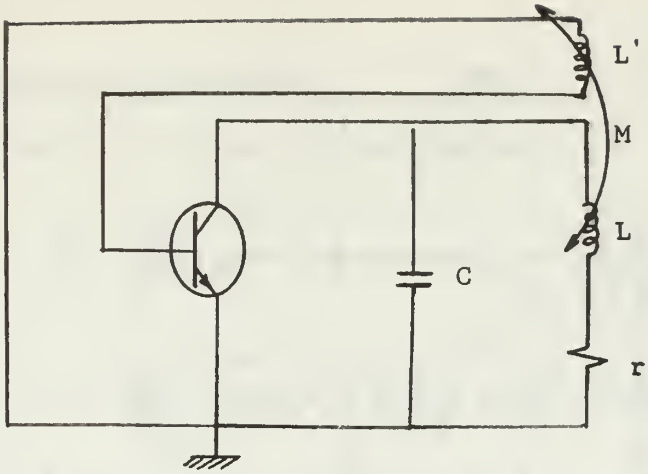


Figure 7-4. Transistor Tuned Collector

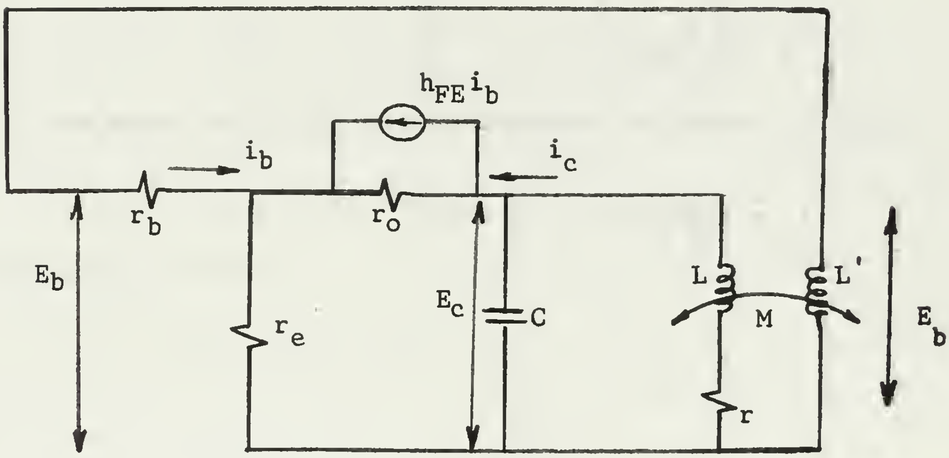


Figure 7-5. Tuned Collector Equivalent Circuit

$$\text{Feedback } B = \frac{E_b}{E_c} = \frac{j\omega M I_L}{-(r + j\omega M) I_L} = - \frac{j\omega M}{r + j\ \ L}$$

$$\text{Voltage amplitude } A_v = \frac{E_c}{E_b} \quad \frac{h_{FE} i_b Z}{i_b \left[r_b + r_e(h_{FE} + 1) \right]}$$

$$\text{impedance } Z = \frac{(r + j\omega L)\dfrac{1}{j\omega C}}{r + j\omega L + \dfrac{1}{j\omega C}} = \frac{r + SL}{LCS^2 + CrS + 1}$$

Characteristic equation: $A_v B = 1$

$$- \frac{SM}{r + SL} \cdot \frac{h_{FE}\, Z}{r_b + r_e(h_{FE} + 1)} = 1$$

$$\frac{(Sh_{FE}M)(r + SL)}{LCS^2 + CrS + 1} = - (r + SL)\cdot\left[r_b + r_e(h_{FE} + 1)\right]$$

$$Sh_{FE}M + \left[LCS^2 + CrS + 1\right]\cdot\left[r_b + r_e(h_{FE} + 1)\right] = 0$$

$$LC\left[r_b + r_e(h_{FE} + 1)\right]S^2 + \left\{Cr\left[r_b + r_e(h_{FE} + 1)\right] + Mh_{FE}\right\}S +$$

$$r_b + r_e(h_{FE} + 1) = 0$$

This is under the assumption that r_o is too large and

$$i_c \doteq h_{FE} i_b$$

8. Tube Tuned Grid Oscillator.

8-1. Derivation of characteristic equation.

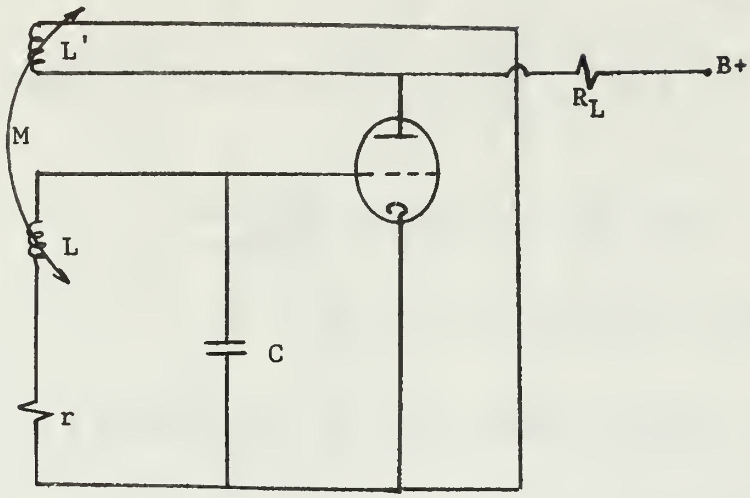


Figure 8-1. Tuned Grid Oscillator

Characteristic equation:

$$y_f Z_f + y_o Z_i + 1 = 0$$

$$y_f = g_m$$

$$Z_f = \frac{M}{CZ}$$

$$Z_i = \frac{\omega^2 M^2}{Z} + j\omega L' = -\frac{S^2 M^2}{Z} + SL'$$

$$Z = r + SL + \frac{1}{SC} = \frac{LCS^2 + rCS + 1}{SC}$$

Plugging into the characteristic equation we get:

$$g_m \frac{M}{CZ} + y_o\left(\frac{SL'Z - S^2M^2}{Z}\right) + 1 = 0$$

$$g_m M + y_o C(SL'Z - S^2M^2) + CZ = 0$$

$$g_m M + y_o C\left[SL' \cdot \frac{S^2LC + SCr + 1}{SC} - S^2M^2\right] +$$

$$+ C \cdot \frac{S^2LC + SCr + 1}{SC} = 0$$

which gives

$$S^3(y_o L'LC - CM^2 y_o) + S^2(L'Cry_o + LC) + S(g_m M + L'y_o + rC) + 1 = 0$$

where

$$y_o = \frac{1}{Z_o} = \frac{1}{r_p//R_L} = \frac{r_p + R_L}{r_p R_L}$$

dividing by y_o we set gain $A = \dfrac{g_m}{y_o}$

$$S^3(L'LC - CM^2) + S^2(L'Cr + \frac{LC}{y_o}) + S(AM + L' + \frac{rC}{y_o}) + \frac{1}{y_o} = 0$$

8-2. Parameter plane curves C versus L' for different gains.

We give different values to the tube circuit parameters such as:

$y_o = 10^{-4}$ mhos

$r = 100\ \Omega$

$M = -10^{-4}$ henry

$L = 10^{-3}$ henry

The characteristic equation becomes:

<u>For A = 30</u>

$$S^3\left[10^{-3}L'C - 10^{-8}C\right] + S^2\left[10^2L'C + 10C\right] + S\left[10^6C + L' - 3 \times 10^{-3}\right] + 10^4 = 0$$

<u>For A = 40</u>

$$S^3\left[10^{-3}\ L'C - 10^{-8}C\right] + S^2\left[10^2L'C + 10C\right] + S\left[10^6C + L' - 4 \times 10^{-3}\right] + 10^4 = 0$$

<u>For A = 50</u>

$$S^3\left[10^{-3}\ L'C - 10^{-8}C\right] + S^2\left[10^2\ L'C + 10C\right] + S\left[10^6C + L' - 5 \times 10^3\right] + 10^4 = 0$$

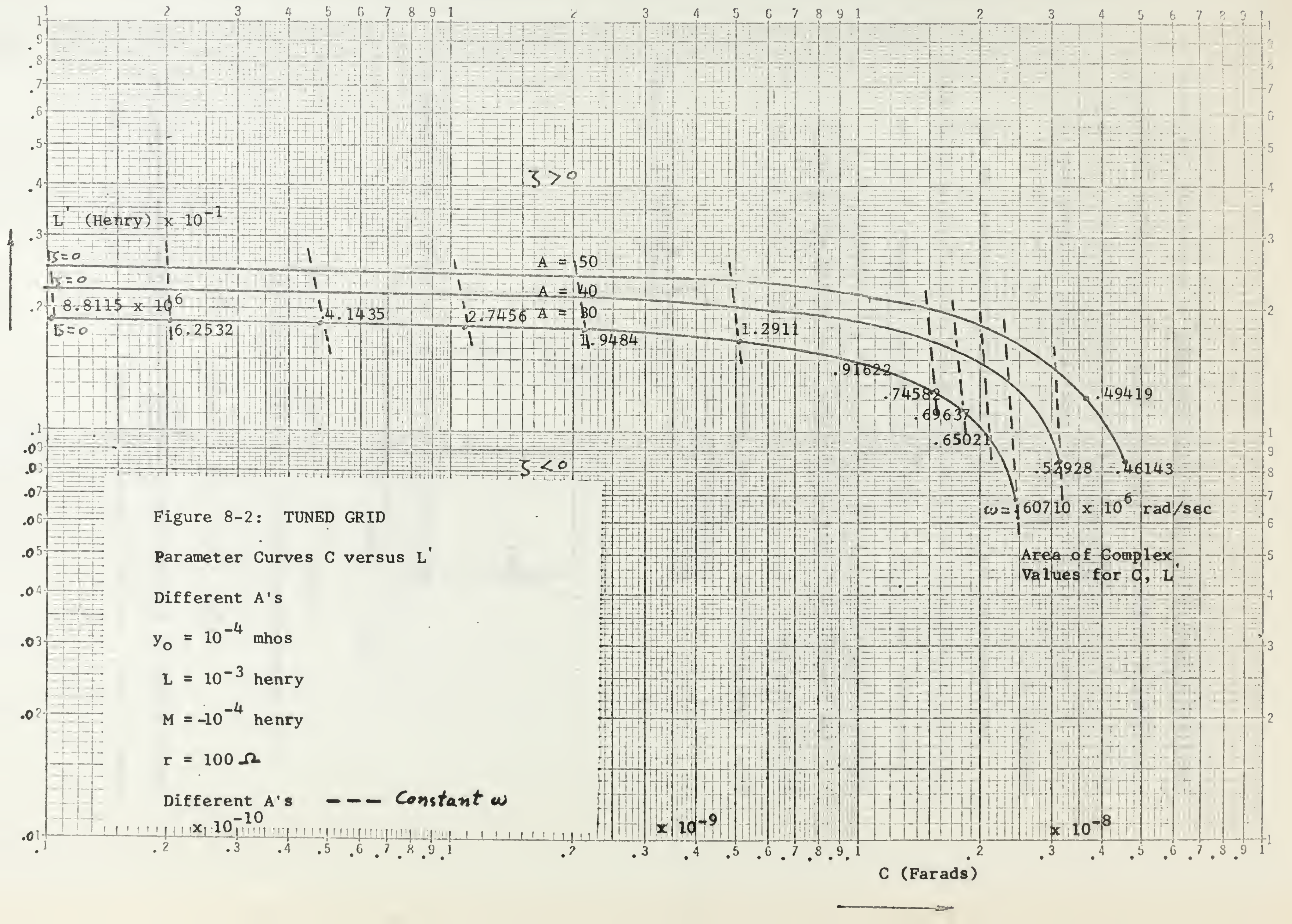


Figure 8-2: TUNED GRID

Parameter Curves C versus L'

Different A's

$y_o = 10^{-4}$ mhos

$L = 10^{-3}$ henry

$M = -10^{-4}$ henry

$r = 100\,\Omega$

Different A's — — — *Constant ω*

Results for Tuned Grid: Parameters C versus L$'$.

From figure 8-2, one notes that the $\zeta = 0$ curves for different gains have the same shape. Namely, for higher frequencies, i.e. $\omega > 1.2 \times 10^6$ rad/sec, the parameter L$'$ remains almost constant with a variation of C.

For frequencies $\omega < 1.2 \times 10^6$ rad/sec, both parameters are required to be varied for oscillations. Beyond a certain value of ω for a given gain, such as $\omega = .6071 \times 10^6$ rad/sec for gain A = 30, the digital computer output gives complex values for parameters C, L$'$.

The constant ω curves are straight lines of the same slope on the semi-log paper.

8-3. Transistor tuned grid (base) oscillator.

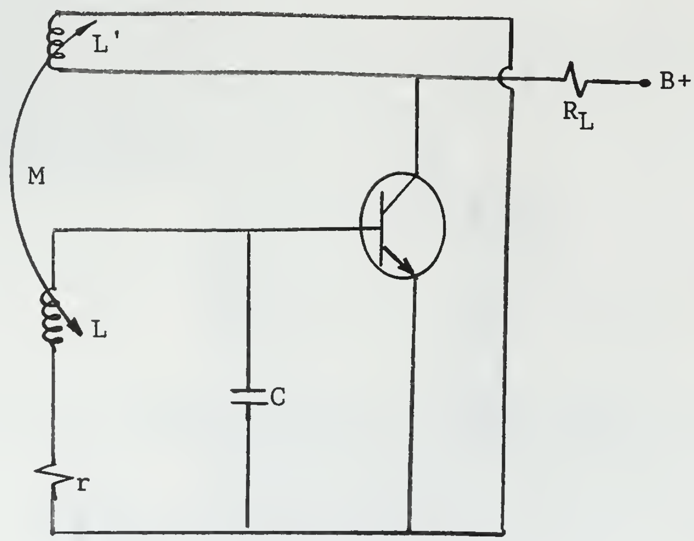


Figure 8-3. Transistor Tuned Base

The characteristic equation of this circuit is:

$$y_f Z_f + y_i Z_o + y_o Z_i + \Delta_y \Delta_Z + 1 = 0$$

and it cannot be further simplified as it could in the tube circuit, because y_i and y_r are not in general too small to be neglected. We will find the y parameters of the transistor in terms of the h parameters and the parameters of the transistor equivalent T circuit which follows:

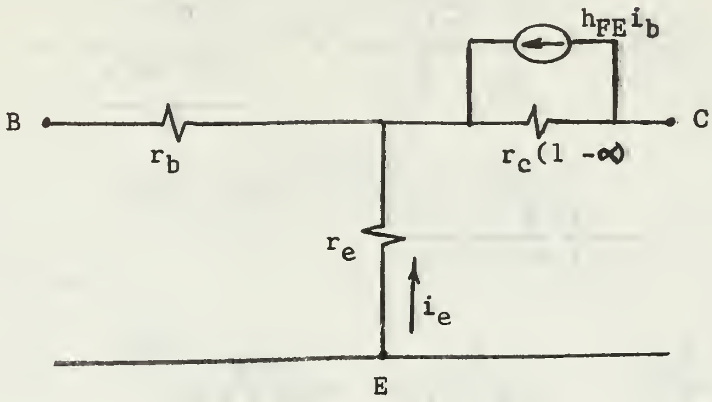


Figure 8-4. Tuned base equivalent circuit

For the circuit above, with a load resistance:

$$y_i \equiv y_{11} \equiv \frac{1}{r_{in}} = \frac{1 + h_{22}R_L}{h_{11} + \Delta hR_L} .$$

The rest of the parameters are:

$$y_r \equiv y_{12} = -\frac{h_{12}}{h_{11}}$$

$$y_f \equiv y_{21} = \frac{h_{21}}{h_{11}}$$

$$y_o \equiv y_{22} = \frac{\Delta h}{h_{11}}$$

h parameters in terms of transistor parameters are:

$$h_{11} = r_b + \frac{r_e}{1-\alpha} = r_b + \frac{\beta r_e}{\alpha} \quad \text{where } \beta \equiv h_{FE}$$

$$h_{12} = \frac{r_e}{(1-\alpha)r_C} = \frac{\beta r_e}{\alpha r_C}$$

$$h_{21} \equiv h_{FE} \equiv \beta$$

$$h_{22} = \frac{1}{(1-\alpha)\, r_C} = \frac{\beta}{\alpha r_C}$$

We now find y parameters in terms of the transistor parameters and α, β

$$y_r = -\frac{h_{12}}{h_{11}} = -\frac{\dfrac{r_e\beta}{\alpha r_c}}{r_b + \dfrac{\beta r_e}{\alpha}} = -\frac{\dfrac{r_e\beta}{\alpha r_c}}{\dfrac{r_b\alpha + \beta r_e}{\alpha}} =$$

$$-\frac{r_e\beta}{r_c(r_b\alpha + \beta r_e)}$$

$$y_f = \frac{h_{21}}{h_{11}} \quad \frac{\beta}{\dfrac{r_b\alpha + \beta r_e}{\alpha}} \quad \frac{\alpha\beta}{r_b\alpha + \beta r_e}$$

$$\Delta h \equiv h_{11}h_{22} - h_{12}h_{21} = \left(\frac{r_b\alpha + \beta r_e}{\alpha}\right)\left(\frac{\beta}{\alpha r_c}\right) - \beta\left(\frac{\beta r_e}{\alpha r_c}\right)$$

$$= \frac{\beta(r_b\alpha + \beta r_e)}{\alpha^2 + r_c} - \frac{\beta^2 r_e}{\alpha r_c} =$$

$$\frac{\beta(r_b\alpha + \beta r_e) - \alpha\beta^2 r_e}{\alpha^2 r_c} = \frac{\beta\alpha r_b + \beta^2 r_e(1 - \alpha)}{\alpha^2 r_c} =$$

$$\frac{\beta\alpha r_b + \beta^2 r_e \dfrac{\alpha}{\beta}}{\alpha^2 r_c} = \frac{\beta(r_b + r_e)}{\alpha r_c}$$

$$y_o = \frac{\Delta h}{h_{11}} \quad \frac{\dfrac{\beta(r_b + r_e)}{\alpha r_c}}{\dfrac{\alpha r_b + \beta r_e}{\alpha}} \quad \frac{\beta(r_b + r_e)}{r_c(\alpha r_b + \beta r_e)}$$

$$y_i = \frac{1 + h_{22}R_L}{h_{11} + \Delta h R_L} \quad \frac{1 + \dfrac{\beta R_L}{\alpha r_c}}{\dfrac{\alpha r_b + \beta r_e}{\alpha} + R_L\beta\dfrac{r_b + r_e}{\alpha r_c}}$$

$$y_i = \frac{\dfrac{\alpha r_c + \beta R_L}{\alpha r_c}}{\dfrac{r_c(\alpha r_b + \beta r_e) + R_L \beta (r_b + r_e)}{\alpha r_c}} =$$

$$\frac{\alpha r_c + \beta R_L}{r_c(\alpha r_b + \beta r_e) + R_L \beta (r_b + r_e)}$$

Z parameters are:

$$Z_i = - \frac{S^2M^2}{Z} + SL'$$

$$Z_r = Z_f = \frac{M}{CZ}$$

$$Z_o = - \frac{1}{S^2C^2Z} \quad \frac{1}{SC}$$

$$\Delta Z = \frac{L'}{C} - \frac{1}{CZ} \left(\frac{L'}{SC} + SM^2\right)$$

$$Z \doteq r + SL + \frac{1}{SC} = \frac{S^2LC + SrC + 1}{SC}$$

Plug these Z quantities into equation

$$y_fZ_f + y_iZ_o + y_oZ_i + \Delta y \Delta Z + 1 = 0$$

and we get

$$y_f \frac{M}{CZ} + y_i\left(\frac{1}{SC} - \frac{1}{S^2C^2Z}\right) + y_o\left(SL' - \frac{S^2M^2}{Z}\right) +$$

$$\Delta y \left[\frac{L'}{C} - \frac{1}{CZ}\left(\frac{L'}{SC} + SM^2\right)\right] + 1 = 0$$

Manipulating this expression, we get:

$$S^3\left[y_oC^2LL' - y_oM^2C^2\right] + S^2\left[y_oC^2rL' + (\Delta y)L'LC + \right.$$

$$\left. - (\Delta y)CM^2 + C^2L\right] + S\left[y_fMC + y_iLC + y_oCL' + \right.$$

$$+ (\Delta y)L'rC + rC^2\Big] + \Big[y_i rC + C\Big] = 0$$

Now put in the y parameters and get the characteristic equation for the tuned base transistor oscillator.

Final expression:

$$S^3\Big[y_o CLL' - y_o M^2 C\Big] + S^2\Big[y_o CrL' + (\Delta y)L'L - (\Delta y)M^2 + CL\Big]$$

$$+ S\Big[y_f M + y_i L + y_o L' + (\Delta y)L'r + rC\Big] + \Big[y_i r + 1\Big] = 0$$

9. Discussion and Conclusions.

The characteristic equations of tube phase shift oscillators have been developed first and formed in such a way that they fit the parameter plane technique, for which a brief summary is given.

Then the phase shift oscillator is studied using three sets of parameters i.e., C_v versus R_v, C_v versus A and C_v versus R_l, for which the $\zeta = 0$ curves for different gains and sensitivity curves have been calculated and plotted.

The characteristic equations of Colpitt's, Hartley, tuned grid, tuned base and tuned plate oscillators have been derived using the general technique given by reference [2] based on two-port network properties.

Colpitt's oscillator is analyzed for parameters C_1 versus L and C_2 versus L and the sensitivity curves have been drawn for different values of tube gains.

The Hartley oscillator has been studied for parameter set C versus L_1, C versus L_2 and C versus A in the same fashion. The digital computer gives complex values of parameters C, L_2 for a certain area ("Dead Zone") and at first glance it seemed that the oscillator would not oscillate in the zone. This phenomenon is further investigated by simulation of the oscillator in this zone by analog and digital computer, and it is found that it actually oscillates. Also, the apparent discontinuity in the C versus L_2 curves is checked analytically, point by point and the curve is shown to be continuous.

Also, the tuned plate and tuned grid oscillators have been studied for one set of parameters, C versus r and C versus L', respectively, for different gains.

In order that the different problems will be computerized, the components of the circuit and the amplifier had been given different numerical values which may or may not occur in an actual circuit, but this has been done for the sake of simplicity. As a consequence, the parameters and/or the frequency of oscillations may sometimes come out in values that usually do not occur in a real oscillator circuit, but it is beyond the scope of the present text, where we are interested in studying only the pattern of parameter variations.

From the phase shift oscillator parameter plane curves C_v versus A, we have found that the minimum gain A for oscillations to be sustained is $A \doteq 28$ as has been proved analytically, so the validity of the technique is reaffirmed.

Also, from the Hartley parameter plane curves C versus A, it is found that the minimum gain to sustain oscillations is A = 1 by proper selection of the other parameter C.

For several cases we notice that one parameter remains constant, while the frequency changes with the other. This result can be used to detect the variation of the other parameter which may represent a quantity to be measured, by just measuring the frequency variation.

The parameter plane technique can be applied to circuits containing not only passive elements but active elements also under the constraint that the active element operates in a linear region.

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The paremeter plane techniques were first introduced in an IEEE paper dated November 4, 1964. The paper dealt mainly with the theory of parameter planes whereby the roots of a polynomial could be determined graphically in terms of two parameters which may appear linearly in any of the coefficients.

Later on methods were developed in manipulating cases with parameters appearing as a product in the coefficients. Also by incrementing a third parameter every time, we can talk about a three-dimensional parameter plane.

In this text parameter plane techniques have been used to plot the imaginary axis as a function of two parameters. From this plot we get curves of frequency versus each of the parameters so that we can study the sensitivity of different types of oscillators.

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